Building mathematical understanding in the classroom: A constructivist teaching approach

A cross-sectoral project funded under the Australian Government's Numeracy Research and Development Initiative and conducted by Catholic Education South Australia

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Executive Summary

A major policy objective of the Australian Government is to ensure that all students attain sound foundations in literacy and numeracy. In 1998 all Education Ministers agreed to a national Literacy and Numeracy Plan that provides a coherent framework for achieving improvement in student literacy and numeracy outcomes. The 1999 Adelaide Declaration on National Goals for Schooling in the Twenty-First Century contains the national literacy and numeracy goal that students should have attained the skills of numeracy and English literacy, such that *every student should be numerate, able to read, write, spell and communicate at an appropriate level.*

In support of the numeracy component of the National Plan, the Australian Government implemented the Primary Numeracy Research and Development Initiative in 2001. This Initiative consisted of two complementary strands-a national project strand and a strategic States and Territories projects strand.

Building Mathematical Understanding in the Classroom: a Constructivist Teaching Approach is one of ten strategic research projects undertaken by State and Territory education authorities across Australia. The purpose of these projects is to investigate a broad range of teaching and learning strategies that lead to improved numeracy outcomes.

The project originated from the action research aspects of the Catholic Education South Australian Office's professional development programmes. Integral to the project was teacher's use of the constructivist learning theory of teaching and learning to support students thinking. Teaching mathematics using a constructivist approach recognises that children construct knowledge from their own experiences. In acknowledgement of the constructivist theory of learning, teachers create learning situations where students use their previous learning to build and develop their understanding of mathematics and numeracy concepts.

Within this context, the Numeracy Team wanted to explore what could be learned from an analysis of mathematical learning in classrooms. The project used a case study approach to document and to analyse the experiences and the environment of effective constructivist mathematics classrooms. The project aimed to explore, "What are the teaching strategies that support all students to improve their numeracy outcomes through the construction of meaningful mathematical understanding? The research addressed the questions:

- *What does a mathematics classroom look like when working with constructivist learning theory?*
- *What strategies do teachers use to facilitate learning in an environment that engages all students in the construction of their own knowledge?*

The research also set out to make interpretive statements about effective numeracy teaching strategies in the primary years, Years 3-5, of schooling and draw implications for teachers by:

- investigating teaching strategies that enable the teacher to identify informal and prior knowledge that individual students bring to their learning; support and encourage students to use this knowledge, as well as their 'natural inventiveness'; and support students to see the links between school mathematics and their world; and
- suggesting a flexible planning framework that is responsive to, and supportive of, individual students' learning needs.

An action research approach using quantitative and qualitative methods was used in the nine mathematics classrooms. The research incorporated quantitative pre-test and post-test measures to establish what learning had taken place, and qualitative methods involving classroom action research and detailed analysis of the action research documentation. Two project researchers



worked collaboratively with ten classroom teacher researchers within nine schools in metropolitan and country regions. Teachers involved in the project worked from the premise that all students are capable of succeeding and that students take different pathways and develop mathematical understandings in different ways at varying rates.

The three teachers and classrooms that achieved the greatest growth in numeracy outcomes were the subject of detailed case studies, describing their classroom interactions, student behaviours and teaching styles. These teachers utilised the Growth Points, that is, a description of patterns of growth in student learning, to plan, design tasks, and assess students' mathematical thinking. The Growth Points were developed by the Catholic Education South Australian Office from a study of Growth Points in linear measurement (Pengelly and Rankin, 1985). The Growth Points also assisted the teachers in understanding the array of connections students might make as they learned about the mathematical concepts of linear measurement and area. The Growth Points evolved into a more formalised framework called *Possible Learning Connections* (see Appendix B). The final chapter of the report summarises the findings of the three detailed cases studies to propose a range of teaching strategies to improve numeracy outcomes for all students.

The key findings of the project are:

- The teaching and learning process is influenced by the social and physical environment where it takes place. The social and physical settings of more effective constructivist mathematics classrooms create a welcoming climate, and encourage cooperative relationships and support learning.
- There is no single aspect of teaching or environment feature that produces learning for all students. Rather, supporting students to think and work mathematically requires the use of a combination of pre-arranged conditions and responsive teaching strategies.
- The "Supportive Conference" model was a major contributing factor in the improvement of all students' mathematics/numeracy. Teachers firstly established each student's current mathematical thinking and understanding using questioning techniques, either with individuals or in small groups. Teachers then continued this interactive process to support students in building their understanding of new mathematical and numeracy concepts.
- Each teacher consciously drew upon a number of factors to affect optimal numeracy learning outcomes. These factors included the teacher's command of mathematics and their use of constructivist learning principles.
- Each teacher drew on the *Possible Learning Connections* framework, describing stages of growth, to support the conferencing. The framework:
  - helped teachers to understand the different pathways students may take to construct their mathematical understandings;
  - provided a tool to support teachers when observing students working, reflecting on students work, assessing students' thinking and planning of units of work;
  - enabled teachers to explore and identify student's existing knowledge in order to build on this knowledge; and
  - supported the teacher in catering for the diverse range of learners and thinking in any classroom.

The research showed that professional learning through collaborative action research supported the teachers to further develop their own knowledge of mathematics, their understanding of students' thinking and how a constructivist approach to mathematics teaching can contribute to students' learning.

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Preamble

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Note: All names of schools and children used in this report have been changed to ensure confidentiality.

Introduction

The Numeracy Team of the Catholic Education Office of South Australia has for many years been using an action research methodology as part of its professional development programs. Teachers have benefited from participating in the programs in being able to reflect on the efficacy of their classroom practices in order to improve students' abilities to think and work mathematically.

Anecdotally, these programs have been successful in supporting individual teachers to refine their classroom practices. Integral to all the action research projects have been constructivist learning theory (as supported in the National Statement on Mathematics for Australian Schools, 1990), the pedagogical assistance provided by the Measurement Growth Points (*see* Appendix A) and the parameters of the more recently developed South Australian Curriculum Standards and Accountability Framework (SACSA) the principles of which draw on constructivist learning theory. As stated in the SACSA document ''…the central idea behind constructivism is that the student is active in the process of taking in information and building knowledge and understanding; in other words, of constructing their own learning. …Within this model of learning educators play a crucial role in encouraging children and students to discover deeper principles and make connections among ideas, or between concepts, processes and their representation. The pedagogy that is best suited to this process is engagement in interactive talk, through which the educator aims to offer, reinterpret and challenge relevant information, based on an assessment of the learner's current state of understanding''.

Further discussion of constructivist teaching and learning practices can be found at http://www.sacsa.sa.edu.au/splash.asp.

With this background, the Numeracy Team wanted to explore what could be learned from the collective action research developments of individual teachers of mathematics. Research questions that started to arise included:

- What does a mathematics classroom look like when working with constructivist learning theory?
- What strategies do teachers use to facilitate learning in an environment that engages all students in the construction of their own knowledge?

The Numeracy Team of the Catholic Education Office were able to do this with Australian Government support through its Numeracy Research and Development Initiative. The project set out to address the question:

What are the teaching strategies that support all students to improve their numeracy outcomes through the construction of meaningful mathematical understanding?

The major part of this report focuses on the research, which melded quantitative and qualitative methods as well as multiple levels of analysis. The final chapter draws on this endeavour to explore what has emerged as pragmatic implications for classroom teachers of mathematics.



Literature Review

The theoretical underpinnings of this research cover many interconnecting themes. These themes are discussed and situated within learning theories and current research. Since the setting for this research was in the classrooms based on constructivist learning theory, it follows then that the literature review centres on a discussion about constructivism that situates this research within those theories and, in particular, social constructivism.

Other interconnecting threads in the review relate to the theme of social interaction, in terms of its role in: students' mathematical acculturation and use of mathematical language; the teacher's influence on those mathematical practices and social norms that are valued in the classroom; and the contribution of reflective intelligence in supporting deeper mathematical understanding.

The theoretical framework

Introduction

To think and work mathematically requires conceptual understanding. By coming to such understandings through their formal education, students are prepared to apply mathematical knowledge and skills when appropriate, to solve new problems, and to better comprehend unfamiliar situations (both human and natural). Research has found that if students do not acquire sufficient understanding they tend to compartmentalise their knowledge and may later experience difficulties when addressing mathematically related problems. In addition to that, if they do not understand, the mathematical skills/knowledge remains within the mathematics classroom, so that there is a struggle to make connections with real life experiences.

Both the Cockcroft Report (1982) and the reports of the National Council of Teachers of Mathematics (NCTM, 1991, 2000) emphasised the need for mathematics teachers to provide students with strategic opportunities to acquire meaning and mathematical understandings. For this to happen there must be major changes to our current thinking about teaching (see, for example, Wood & Turner-Vorbeck, 2001; Cobb, Yackel & Wood, 1992). In response to these reports, researchers in mathematics education are questioning traditional ideas and practices of mathematics teaching and learning and changing to a problem solving approach where students not only learn basic skills, but also develop higher order thinking.

Taking the view that mathematics is not static but rather a humanistic field that is continually growing and reforming, and that children construct their own knowledge (Hersch, 1997), then teaching can no longer be a matter of viewing students' minds as 'empty vessels' ready to adopt, internalise and reproduce correct mathematical knowledge and applications. Rather, we have come to learn that teaching which includes instructional contexts where students are supported to move from their own intuitive mathematical understandings to those of conventional mathematics, produces more profound levels of mathematical understandings (Skemp, 1971). In the past, research about the teaching and learning of mathematics tended to have a psychological inclination, which required the teacher/instructor to understand the students' mental processes, and to use those mental processes to facilitate teaching and learning. It was not until recently that researchers (for example, Carraher et al., 1985; Lave, 1988; Wood & Turner-Vorbeck, 2001; and Bauersfeld, 1988) became engaged in researching the sociological aspects of teaching and learning mathematics.

To understand what count as effective teaching strategies, we need to look at them in conjunction with students' activities as a form of social interaction; investigate the



different aspects of teaching and learning and where the learning takes place. Our research will therefore recognise the psychological aspects of learning but also the need to ‘acculturate’ the next generation, as well as investigating the mathematical environment the teacher creates to support the students as they construct their understandings of mathematics.

Constructivism

This research has used constructivist learning theory when investigating all of the classrooms. It chose to do so because all South Australian Catholic schools are using SACSA framework, which is in turn based on constructivist theory. The ‘central idea behind constructivism’ As stated in SACSA framework document, “*…the central idea behind constructivism is that the student is active in the process of taking in information and building knowledge and understanding; in other words, of constructing their own learning”.* Within this model of learning educators play a crucial role in encouraging children and students to discover deeper principles and make connections among ideas, or between concepts, processes and their representation. The pedagogy that is best suited to this process is engagement in interactive talk, through which the educator aims to offer, reinterpret and challenge relevant information, based on an assessment of the learner’s current state of understanding”.

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It is widely recognised by mathematics educators and philosophers alike that different forms of constructivism exist, for example, radical and social constructivism.

Radical constructivism

Radical constructivism, as described by Von Glaserfeld (1994), is based on neo-Piagetian foundations. Like Piaget, Von Glaserfeld has a biological foundation to his theory, with individuals constructing knowledge through a process of cognitive adaptation in terms of a learner’s assimilation and accommodation of perceptions. He describes the learner as growing and incorporating past experiences into current ones, thus emphasising the individualised orientation to learning. According to Von Glaserfeld, radical constructivism is based on two tenets:

- Knowledge is not passively received, but actively built up by the cognising subject.
- The function of cognition is adaptive and serves the organisation of the experimental world, not the discovery of ontological reality.

These tenets point out that knowledge is not transmitted from the teacher to the students but rather the students have to construct their own knowledge through their interpretation of the world as created by the teacher. In this case both the teacher and the students are reconstructing the knowledge to come to a common understanding (Wertsch, 1985; Vygotsky, 1978; and Edwards & Mercer, 1991).

Standing on its own, the first tenet is criticised as weak constructivism as it is too simplistic (Lerman, 1989). The second tenet indicates that knowledge is subjective and that the learner has a major input in shaping the learning. Additionally this tenet recognises that knowledge is not a ‘set body of wisdom’ waiting to be discovered, but is constructed and added to or changed.

Analysing the theory of radical constructivism, Ernest (1991) says that what has made radical constructivism attractive is its refutation of absolutism in epistemology, which he analyses as being associated with behaviourist and cognitivist learning theories.

Radical constructivism, with its emphasis on the individual's own learning experiences, has been heavily criticised by many researchers, including Bruner (1990) and Confrey (1994), for omitting to consider the broader aspects of mathematics learning. For example, Confrey points out that radical constructivism appears to overlook the social aspects of learning. She goes on to explain that an individual's experiences of the world occur within a social context and meanings are influenced by this social situation.

Confrey states that

> The constructivist relies on explanations based on the interplay between the social negotiation of meaning and individual creativity. (Confrey, 1994, p. 114)

She makes it clear that the social context may even dictate what meaning the child constructs. Following in the same vein, Solomon (1989) argues that understanding occurs within a social environment. Solomon (cited in Goodchild, 2001) notes that "knowledge is intrinsically social". He discusses how one builds understanding of an object, emphasising that it entails knowing the social practices of how the object is used. Both Solomon and Confrey outline the important framing influence of the social context as the individual constructs knowledge.

Social constructivism

Like radical constructivism, social constructivism is based on a cognitive theory, namely that of Vygotsky (1986). Vygotsky proposes that knowledge is not constructed individually, but happens through the internalisation of social knowledge, which is embodied in social and cultural practices. Based on this cognitive theory, social constructivist theory acknowledges the active construction of knowledge formed by the learner on the basis of experiences and prior knowledge. It places major importance on the use of language and the role of interactions with others and their physical world as contributing factors to the construction of knowledge.

However, both Ernest (1991) and Lerman (1996) identify a number of problems associated with social constructivism that need to be resolved. They both recognise as problematic that social constructivism takes the principles of radical constructivism and puts them together with 'socio-cultural' psychology to give a single cognitive theory. Cobb (1994) believes that a socio-cultural perspective of learning cannot be attained in this way. He argues that the 'mind' could be lost within the social milieu. Constructivism, he stresses, acknowledges that the 'mind' is situated within the individual.

Different approaches have been put forward to explain social constructivism. Cobb (1994), for example, asserted that both theories could cohabit and that teachers can use the strength of both when organising teaching experiences for students. Working from this premise many researchers (Bauersfeld, 1988; Voigt, 1985; Cobb, 1994; and Cobb, Wood, & Yackel, 1990) incorporate social interactions into their definitions of constructivism. These researchers place the individual as central in the meaning-making process but have developed justifications for how the environment, including other people, play a part in the construction of personal meaning.

This study has taken Cobb's (1994) approach of constructivist theory as a theoretical foundation for supporting the learning and understanding of mathematics. It acknowledges the individual as s/he constructs and makes meaning, as well as the social environment, which also forms a crucial and active role in assisting the learner when



constructing mathematical knowledge. It acknowledges the social domain and its influences on the development of the individual as s/he constructs meaning in response to these external stimuli within the social context.

Developing mathematical understanding and numeracy

The construction of mathematical understanding

Fathoming how we learn mathematics with understanding has occupied theorists and researchers alike for many years. Skemp's (1989) theory on the psychology of learning mathematics identifies the importance of learning mathematics with understanding, which he describes as reflective intelligence. He points out that individual schemas are very important in the formation of conceptual structures. If the early schemas were inappropriately made the student may later have difficulties with assimilation of more complex ideas. In his argument Skemp points out that schemas that are well formed are long term. These schemas take into account the bigger picture, not just the immediate task. This leads to what he calls instrumental and relational understanding. He argues that

> Logical understanding is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning. (Skemp, 1989, p. 45)

So if students have well formed schemas they are able to combine relevant mathematical ideas into logical reasoning, creating a network of ideas that could be called upon and used appropriately.

> Focusing on creating a network of ideas, Hiebert and Carpenter (1992) suggest that when learning with understanding occurs, a mathematical idea, procedure, or fact that is understood thoroughly is linked to existing networks with stronger or more numerous connections [where] mental representations are enriched by being connected with a network of ideas. (Hiebert & Carpenter, 1992, p. 67)

The growth of mathematical understandings involves students in the construction and/or assimilation of new schemas into existing schemas in order to create a network of reference points, which they in turn use to connect and give meaning to new ideas. In order for students to construct, link and assimilate their schemas they must be located in social settings where they can construct useful and powerful connections.

Mathematics is presented and communicated with symbols and words. The symbols system has been invented to model meaning precisely. Skemp (1986) argues that students can, however, operate at the superficial level only; that is, they can use the symbols without entering into their meaning. Skemp also stresses the necessity for students to delve into the semantics of mathematics if meaning and understanding are to be constructed. Communication, therefore, seems to play a major role in influencing understanding in mathematics. Vygotsky's theory asserts that an individual's thinking is formed through internalised conversations devised from interactions with others.

Advocating the perspective that language is a powerful factor in the accommodation of individual and collective thinking, Vygotsky (1985) argues that an individual constructs his/her own understanding through the use of language and social interactions. He describes a child's cultural development as appearing on two planes: firstly, on the social plane between two people as an interpsychological category, and then on the psychological plane, that is within the child as an intrapsychological category. What happens within the intrapsychological stage

supports (scaffolds–*see* Wood, Bruner & Ross, 1976) the student to move into the interpsychological stage where the student accommodates the learning into existing schemas, creating a network of ideas (Hiebert et al., 1997). There is an emerging consensus recognising that schemas and networks relate to how students construct and retain knowledge, that some are more coherent than others, and that effective mathematics understanding depends on how teachers recognise and accommodate this in their pedagogy. This is, however, not the whole story, as we now need to look at the role of metacognition in the construction of mathematical understanding.

Understanding through reflection/metacognition

In 1963 Dienes studied a class of young children using manipulatives when constructing understanding of arithmetic. Dienes (2002) commented on his findings, suggesting that, if a child who is learning arithmetic manipulates objects, he is using his sensory intelligence (in Skemp's term), but when the child realises that there is a relationship between his actions and the output, he is using what Skemp (1989) described as 'reflective intelligence'. Skemp (1971) argues that

> When a child can transcend [sensory stimuli], and give responses that indicate his perception of the number property independent of configuration, we may say that he shows sensory intelligence, that is to say, awareness of certain relationships between sensory stimuli. (Skemp, 1971, pp. 47-48)

This then leads into another very important aspect of learning mathematics with understanding–the reflective nature of learning.

Skemp (1982) also recognises the powerful influences of communications in the development of reflective intelligence. Reflection, he argues, occurs when one interacts with other people, resulting in agreement and communal ideas, with the individual becoming less egocentric in the process. Researching students' patterns of interactions with a social context, Hoyles (1982) found that during interactions, especially during a conflict, students were seen to distance themselves from their personal thoughts while they tried to accommodate what others were suggesting. As discussion necessitates the clarification of one's own ideas in order to justify them to others, it may also require the learner to recognise weaknesses, which may involve modification of one's cognitive structure (Hoyles, 1982; Skemp, 1982). To be able to reflect on one's own thinking, one has to be able to know what one knows (Vygotsky, 1978; Wertsch, 1985). Schoenfeld (1992) describes metacognition as three related and yet distinct categories when he states:

> A reasonable hypothesis is that in any situation which requires learners to formulate ideas explicitly, and to justify them by showing them to be logically derivable from other and generally accepted ideas, would exercise and develop the ability to reflect on their schemas. In other words, argument and discussion are useful ways of learning to reflect. (p. 347)

Working with young children who were engaged in problem solving, Carpenter and Hiebert (1993) found that by providing opportunities for students to talk about how they solved the problems, the students learned to reflect on their strategies and findings and also to articulate their thinking. Children have been seen to talk to themselves if they do not have opportunities to discuss their thinking with others as they plan their next action. Children need to consciously



reflect on their thinking in order to use appropriate strategies when solving other problems.

Goodchild (2001) describes metacognition as the learner's ability to reflect upon her own knowledge, control her actions and to plan her next action. Underhill (1991, p. 230) talks about reflection as being the main factor that stimulates cognitive restructuring when there is conflict between the learners or between the learner and the environment, such as the task. This restructuring is informed by the student's experiences and prior knowledge, thus placing the student at the control of his/her own learning. In his work with young children, Baroody (1987) found that when children were allowed to bring in their informal knowledge to the classroom, they were more likely to link their prior experiences of mathematics to classroom mathematics so that their informal knowledge provided these students with a stronger base on which they could build. Thus, they were more likely to understand the mathematics.

Wood and Turner-Vorbeck (2001) assert that within a social interaction the sorts of questions the teacher asks make demands on students to engage in complex thinking and reasoning. The teacher's questions could be complex and therefore involve the students in reflective mathematical activities:

> From these findings, we can link increases in students' participation to the norms constituted and the social interaction, including language that evolves in accordance to sociological theory. Therefore we can begin to understand the interrelationship between the differences in teaching as revealed in participation structures created and processes by which individual and common meaning evolve, progress, or both, during discussion. (Wood & Turner-Vorbeck, 2001, p. 191)

Bruner (1990) emphasises the role of metacognition in thinking, but believes that the development of children's capacities for reflective thought originates in social interactions with others. He says that children need to develop a system of shared meanings in order to participate successfully in human interaction. In order to share common understandings they need an implicit knowledge when speaking to one another. Learning with understanding entails students constructing their own understandings through reflection. They also need to be conscious of what they know and in which situations they could use this knowledge effectively.

Contentions over the meaning of 'Numeracy'

There are continuing debates within the educational community and about the links between mathematics and numeracy. In Australia, State and Territory education authorities have a range of approaches to numeracy education in schools which are reflected in a variety of programmes and initiatives for the development of students' mathematical skills and numeracy (DETYA, 2000). Yasukawa et al. (1994) describe numeracy as the bridge between the broader study of mathematics and the specific context in which it is required. Nunes and Bryant (1998) emphasised that

> to teach mathematics to children in a way that makes all children numerate in today's (and even in tomorrow's) world we have to know much more about how children learn mathematics and what mathematics learning can do for their thinking. As societies change, the concept of what it is to be numerate and literate also changes. (Nunes & Bryant, 1998, p. 2)

Other researchers, such as Nickson (2000) and Clements and Battista (1990), describe the most effective mathematics classroom as one where learners develop understanding and construct their own mathematical knowledge so that they are continually inventing new ways of thinking about their world.

Integral to supporting all students to develop their numeracy skills is to recognise and respect their cultural backgrounds and to build onto each student's existing informal knowledge as well as giving students opportunities to construct their own meaning of mathematics using different possibly culturally aligned strategies. Aubrey (1993, 1994), Hughes (1998), Young-Loveridge (1989), Nunes and Bryant (1998), and Mboyiya (1998) have all reported that students arrive at school with a varied range of numerate abilities and viewpoints that have evolved from their social and cultural contexts. Donaldson (1990) has emphasised the importance for teachers

> to be clear, not only what they would like children to become under their guidance, but about what children are actually like when the process begins. (Donaldson, 1990, p. 15)

Research findings indicate that to support effective mathematics learning, opportunities need to be created for children to construct their own mathematical meanings by reflecting on their existing network of ideas and integrating these with their new found thinking from a range of experiences. These experiences support students to see connections between contexts, integrate an appropriate collection of mathematical skills, knowledge and processes, and transfer them across learning areas as well as beyond the school environment.

Our description of numeracy involves much more than the mathematics classroom can offer. It also acknowledges the contextual and cultural nature of numeracy well beyond the mathematics classroom. We also believe that as numeracy is dynamic and embedded in a rapidly changing society, there is a need to acknowledge that the aspects of numeracy that are currently emphasised as important will change over time. Numeracy, then, can be a means of empowering students to participate in, and contribute to, the society in which they live as part of a rapidly changing world. However, for the purpose of this research we have narrowed the focus to the role of the mathematics classroom in supporting students' numeracy.

Learning mathematics and classroom cultures

Psychological theories seem inadequate when discussing learning within a social environment, as they do not take into account the social and cultural dynamics of the situation within the mathematics classroom. On the other hand, sociological theories seem equally as problematic because they do not account for the individual construction of knowledge. Working from a constructivist perspective, Rogoff (1994, p. 70) has emphasised that ''development is made up of both individual efforts or tendencies and the larger socio-cultural context in which the individual is embedded''. This argument focuses on the individual making sense of the mathematics supported by a socio-cultural environment, which, in this case, manifest in the classroom.

Bruner's (1990) proposition about a cultural psychology perspective also focuses on the relationship between the individual and the social processes of making meaning within a set culture. Bruner emphasised that the meaning one makes is related to the larger context where the



meaning is being made. So the classroom culture exerts a powerful influence on the way the students perceive the mathematics to be learnt and how they see themselves as mathematicians. The teacher's attitudes and beliefs (see teacher input below), as well as other factors, have a great influence on the attributes of the classroom culture.

Vygotsky's theory of cognitive development emphasises the idea that knowledge is social and cultural. An individual's cognitive development is therefore shaped by the internalisation of the social interactions and cooperative activities that occur in social practice. Cobb, Yackel and Wood (1993) suggest that mathematical beliefs and understandings are continually being negotiated through social interactions that are institutionalised by a community of 'knowers'. They assert that:

> students' mathematical learning is influenced by both the mathematical practices and the social norms implicitly and explicitly negotiated and institutionalized by the classroom community. (Cobb, Yackel & Wood, 1993, p. 45)

Solomon (1989) also claims that students' mathematical developments have a social as well as a cognitive aspect. These developments are often constrained by the beliefs and expectations of teachers, schools and the community. Within this environment both the student and teacher are making an interpretive judgement of what is happening and this judgement subtly influences the curriculum. Ernest (1991) discusses how the beliefs about mathematics are communicated through what happens within the classroom.

A socio-cultural perspective of cognition emerges from the work of Lave (1988), Carraher et al. (1985) and Carraher (1991). Following a study of shoppers, Lave argued that the study of cognition must take place in 'everyday' socio-cultural practices, and that 'cognition is situated'. She insisted that cognition could not be studied in isolation, as the subjects' responses depend on the context. It is therefore essential that schools take into account the social aspects of learning and teaching. In the current study, researchers analysed the social and physical settings to determine their forms and their effects on learning and teaching. As the negotiations of mathematical meaning are set within social and physical setting contexts, and teaching is better observed in action, it was essential to look into classroom settings to examine the ways teachers contribute to students' construction of mathematical understandings.

Teachers' dispositions and their possible impact on students' learning

Within the social constructivist classroom, the teacher's role as facilitator of the learning is crucial. Debates about teacher effectiveness and effective teaching strategies have preoccupied researchers (for example, Calderhead, 1987; Smyth, 1987; Schön, 1983; Cobb, 1994; and Wood et al., 1993) for some time. Addressing effective teaching strategies within the social arena, namely the classroom, researchers acknowledge that there exist many variables that could contribute to effective teaching. However, three major influential factors seem to command researchers' attention: teachers' knowledge, attitudes and beliefs.

Shulman (1987) identified three forms of teacher knowledge, which he described as content, pedagogic and curricular. Content knowledge is the knowledge of the subject matter, while the pedagogical knowledge is the knowledge of how to teach. The pedagogical-content knowledge (curricular) is the knowledge of how to teach a concept, which could be specific to that concept only. Shulman (1987) emphasises that pedagogical knowledge goes beyond content

knowledge; it includes an understanding of the pathways different students of different ages and cultural backgrounds take during their learning experience.

Mewborn (2001) argues that having the mathematical knowledge is an important prerequisite for its teaching. She went further to point out that for teachers to be able to teach the content knowledge effectively, they need to have conceptual understanding of the mathematics. Working with primary and kindergarten teachers, Clarke and Cheeseman (2000) and Shifter (1997) also found that content knowledge and conceptual understandings of mathematics are general characteristics of effective teachers. Shifter (1997) further asserts that teachers need to be able to assess what students are saying and the validity of their thinking, and to identify conceptual issues students may have as they construct understanding. Within a constructivist environment, where students are working together to construct their own knowledge, teachers need to be able to assist students in being able to understand their thinking. Carpenter and Fennema (1992) point out that, apart from content knowledge, teachers also need to have the ability to know how children think within a mathematical topic. Therefore if teachers are to provide effective support, they also need to follow individual thinking and to respond to this thinking, keeping the focus on the individual as well as the group.

Addressing teachers' attitudes, Perry and Howard (1999) argue that the pedagogy used in the classroom is determined by the philosophies the teacher holds about mathematics–whether the teacher sees mathematics as a static body of knowledge or as a way of representing the world or possibly as a mixture of both. This means that the teacher's beliefs of mathematics has great impact on the teaching and learning of mathematics in the classroom. Ernest (1989) also discusses the difference between the pedagogical model the teacher advocates about teaching and learning and what really takes place in the classroom and at the teacher's 'level of consciousness'. Cobb (1986) argues that teachers' beliefs about mathematics and the learning of mathematics impinge on the students' beliefs and goals within the subject area. This shows that teachers' beliefs and attitudes about mathematics largely shape the pedagogy they use and hence the responses they obtain from their students. Thus, while mathematical knowledge is an important factor for effective teaching, pedagogical knowledge and the beliefs the teacher holds about mathematics and mathematics learning have even greater implications for effective teaching.

As the teachers participating in this research project were working with the SACSA curriculum framework, which is based on constructivist learning theory, all of the classrooms being researched were either using a constructivist approach or were willing to adopt such approach. While some attention was given to content knowledge, the research concentrated on the pedagogic knowledge and its possible implications for teaching. Using a social constructivist framework and keeping the individual student at the forefront, we reviewed what it means to develop mathematical understanding and subsequently numeracy. We found that the psychological aspect of learning was not sufficient when investigating effective teaching, as teaching and learning happen within a social arena–the classroom; hence the need to include a sociological perspective. The teacher's pedagogical knowledge, beliefs and expectations, also contributed to the students' beliefs and expectations of mathematics.

The research project looked at how primary school students build understanding in mathematics working in constructivist environments and improve their numeracy outcomes. To do this we looked closely at the pedagogy used by the teachers; how they organised the mathematical experiences

to facilitate social as well as individual construction of knowledge and how teachers negotiated the social norms to facilitate social interactions and the negotiations of individual mathematical meanings and understandings for all students within mixed ability classes.

The literature we reviewed was therefore persuasive in shaping the research question of our collaborative design:

> What are the teaching strategies that support all students to construct mathematical understanding and improved numeracy outcomes?

This complex question required a multifaceted naturalistic design. Chapter 2 is a description of the research methodology that we used to address the question.

Research Design and Methodology

The purpose of this chapter is to explain the research design and provide justifications for the methodology used.

Overview of the research

This research project was a collaborative undertaking involving a mixture of quantitative pre- and post-test measures to establish what learning had occurred, qualitative methods involving classroom action research, and a detailed analysis of the action research documentation. Two project researchers worked collaboratively with ten classroom teacher researchers across nine school sites in metropolitan and country regions.

The research comprised two distinct phases:

Phase One was conducted over a period of 11 months from February to December in 2001. It involved the fieldwork, which comprised the pre-tests and post-tests and the action research component of the fieldwork. The year culminated in teachers' case records and an initial analysis of field notes.

Phase Two occurred over the following 18 months from January 2002 to June 2003. This phase involved a subsequent analysis of the fieldwork case records and observations from the action research–firstly to produce three single case analyses which were then reanalysed to produce a combined case analysis.

Details of these two phases follow.

Phase 1: Fieldwork

Phase 1: Quantitative research methods

The quantitative methodology involved all students participating in a pre and post, pencil-and-paper test, as well as two practical observation tests at the beginning and at the end of one school year.

Our concept of numeracy for purpose of the project recognises its contextual and cultural nature beyond the mathematics classroom, where students can call upon, adapt and apply their knowledge of mathematics to solve problems. Therefore all students were tested, using a pencil-and-paper test and a practical test, in March and again in November, 2001.

The test data, for both the pencil-and-paper tests and the practical observation, was analysed using a Rasch modelling. The data for each student was entered into SPSS (Statistical Package for the Social Sciences). The following analysis comes from the written report.

Pencil-and-paper test

The same 24-item pencil-and-paper test was used as both pre-test and post-test, and included validated test questions from the NSW Basic Skills Tests (BST) and the Western Australia Literacy and Numeracy Assessment (WALNA) tasks. Where items involving a particular concept or skill were not available, the researchers devised and validated their own. The Year 3, 4 and 5 test papers were different but had common items. The tests were administered under examination conditions and students were allowed up to 60 minutes to complete them, although most students required only 45 minutes. The questions were read to those students who had been identified as having special needs.

A single Numeracy scale was then constructed across the three year levels using the common items linking procedure. The pattern of the common items across the three tests is set out in Figure 1.

Figure 1. Common items across Years 3, 4 and 5 in the pencil-and-paper Numeracy Test

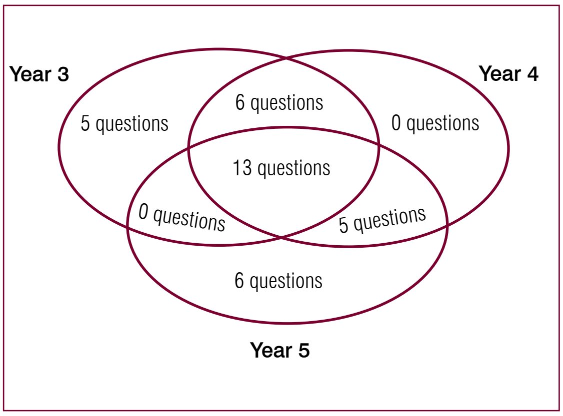


Table 1. Numbers of boys and girls in Years 3, 4 and 5 who participated in the Numeracy Research Tests

Gender	Year 3	Year 4	Year 5	Total
Boys	40	43	57	140
Girls	37	37	55	129
Total	77	80	112	269

Of the 269 students, 42 were Language Background other than English (LBOTE) students. These students were distributed across Year 3–5 classes as shown in Table 2.

Table 2. Numbers of LBOTE Students at each Year Level participating in the Numeracy Research Tests

	Year 3	Year 4	Year 5	Total
LBOTE students	19	7	16	42

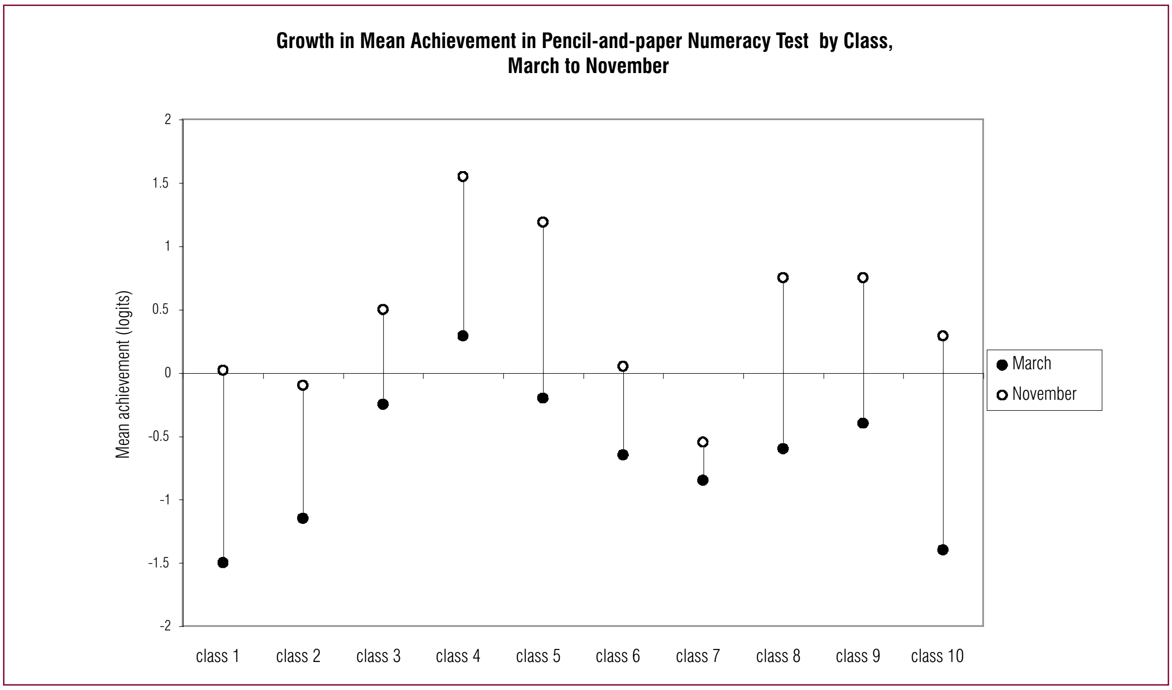
Test analyses

Each student participating in the project in March was given an ID number. Student responses to each item on the two test occasions were matched to this ID number and then entered into SPSS. Once all the data had been entered and checked thoroughly, the tests were examined with the Rasch scaling procedure to determine their validity, reliability and robustness. The advantage of the Rasch analysis is that test items and student responses can be considered together on the same scale, yet they are independent of each other.

The analysis for the pencil-and-paper data resulted in a single Numeracy scale based on the linear, area and angle measurement items. Sub-tests were constructed across the three year levels using the common items linking procedure. This scale was then calibrated using the Rasch scaling procedure to produce a logit (equal interval) scale ranging from –0.5 to +0.5. The November scores were anchored to the March scores, that is, they were scored to exactly the same scale so that direct comparisons could be made.

The mean achievement in numeracy in March and November for each class is presented in Figure 2. The logit scale is presented along the vertical axis, with classes along the horizontal axis. Overall, it is clear that from March to November students in all classes made gains, but these gains were much greater in some classrooms than in others. The highest mean value for both March and November is evident for class 4, while class 7 made the smallest gain.

Figure 2. Growth in mean achievement by class in pencil-and-paper Numeracy test, March to November.



Practical Observation Task

The **practical Observation Task** involved two different contextual investigations. The same two investigations were administered in March and November 2001 to all ten classes–a total of 235 students. One of these investigations required application of linear measurement and the other involved the measurement of area.

The tests were administered under examination conditions outside of the classroom environment. The tests were conducted on a one-to-one basis over a 20-30 minute period for each of the investigations. Each problem was presented verbally to the student and an array of materials was provided for the student to choose from. The investigations were implemented on two different days.

Moderation of observers' marking

The six observers for the pre- and post-test implementation met weekly for approximately 2-3 hours to moderate the process. Each observer rated student responses to the tasks using a scoring rubric developed by Catholic Education South Australia. The tasks and the scoring rubric were set out on a score sheet, which was used by all observers on both occasions.

Analysis of test data

After the data for each student had been entered into SPSS, concurrent files were created by combining the data for each student from the two occasions. Each subscale within the Task and Observation Schedule was then calibrated using the Rasch scaling partial credit model. The Task and Observation Schedule was divided into Linear Measurement

and Area and then into Communication, Problem Solving and Measuring in Units. The five aspects were used to form five subscales for the Task and Observation Schedule.

Table 3. Numbers of boys and girls in Years 3, 4 and 5 who participated in the Practical observation Research Tasks

Gender	Year 3	Year 4	Year 5	Total
Boys	22	43	56	121
Girls	26	36	52	114
Total	48	79	108	235

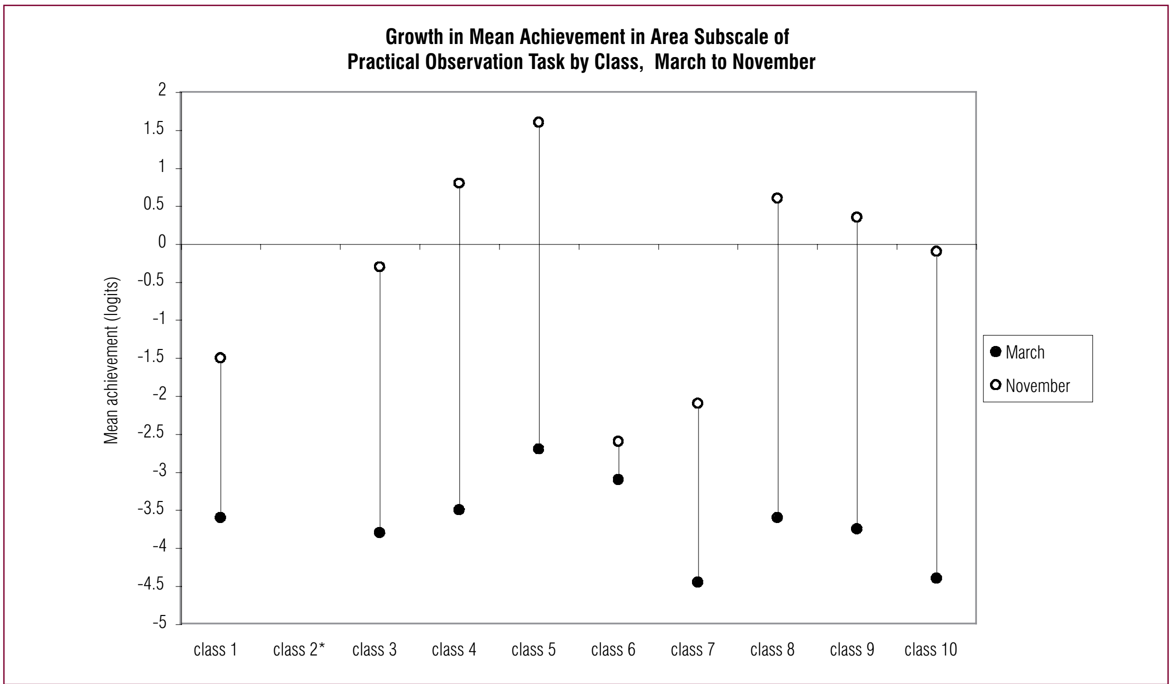
Of the 235 students participating in the practical tasks, 35 were LBOTE students. These students were distributed across Year 3-5 classes as shown in Table 4.

Table 4. Numbers of LBOTE Students at each Year Level who participated in the Practical observation Research Tests

	Year 3	Year 4	Year 5	Total
LBOTE students	12	7	16	35

The mean achievement values for each of the five subscales for March and November for each school are presented in Figures 3 to 7 respectively, for Area, Linear Measurement, Communication, Problem Solving Strategies and Measuring in Units. In each figure, the schools on the horizontal axis were coded as for the Numeracy Tests, and the logit scale represented on the vertical axis.

Figure 3. Growth in mean achievement by class for the Area subscale of the Task and Observation Schedule, March to November



* Class 2 was withdrawn from the project.

Figure 4. Growth in mean achievement by class for the Linear Measurement subscale of the Task and Observation Schedule, March to November

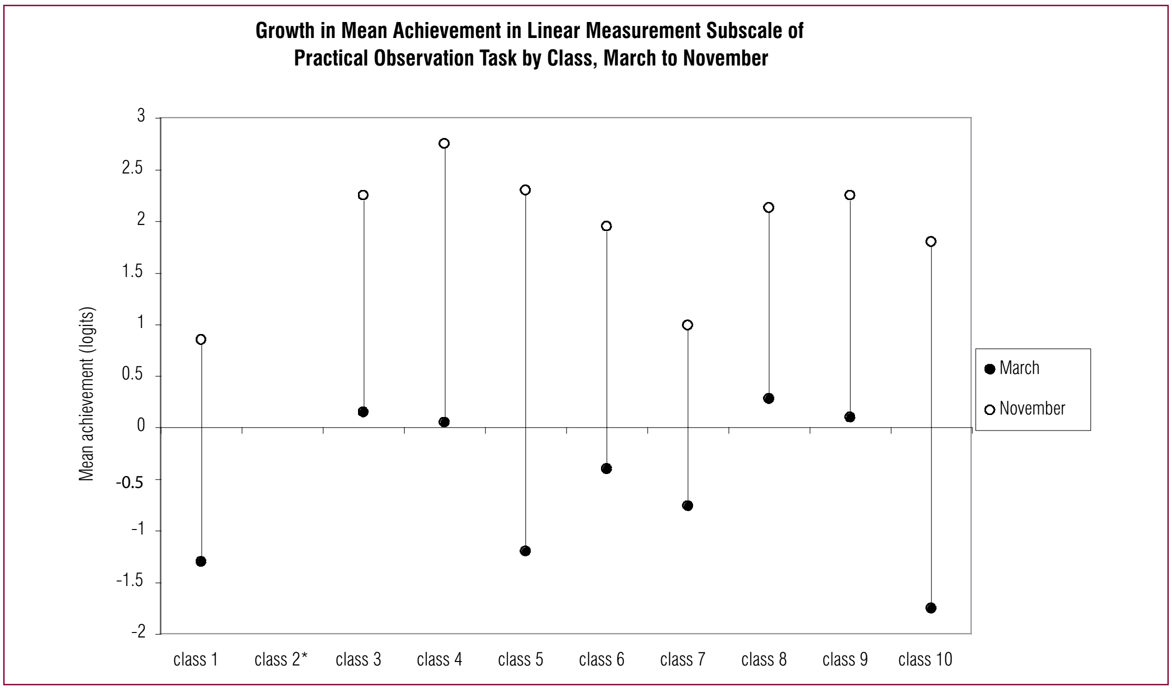
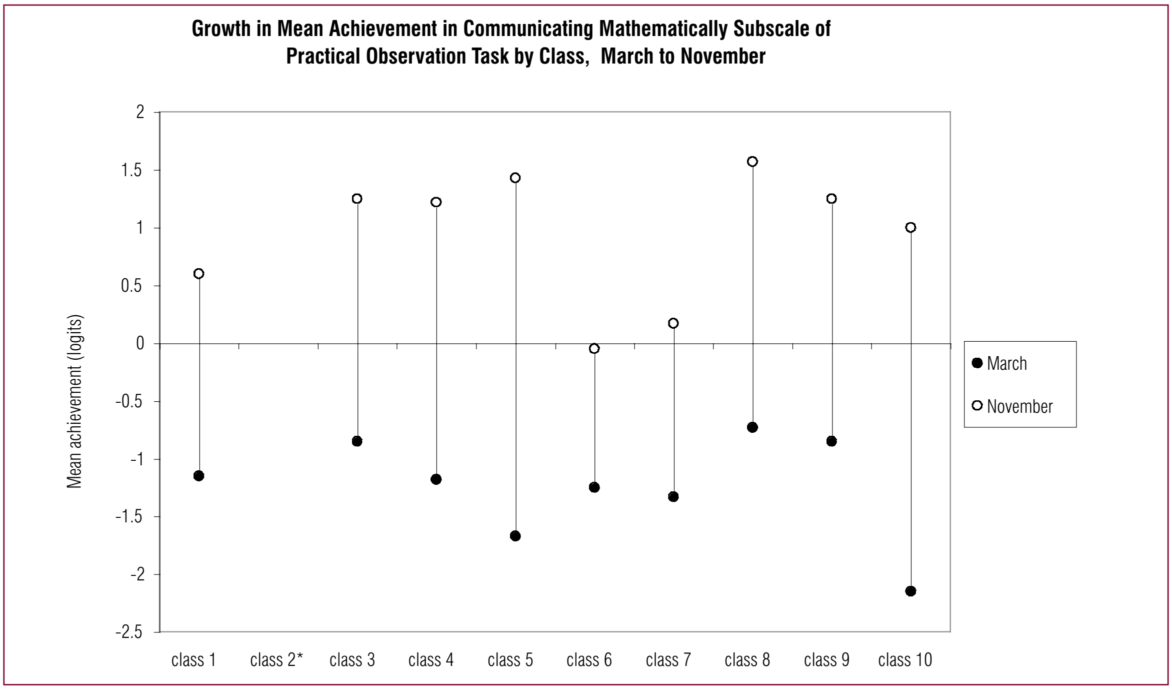


Figure 5. Growth in mean achievement by class for the Communicating Mathematically subscale of the Task and Observation Schedule, March to November



* Class 2 was withdrawn from the project.

Figure 6. Growth in mean achievement by class for the Problem Solving subscale of the Task and Observation Schedule, March to November

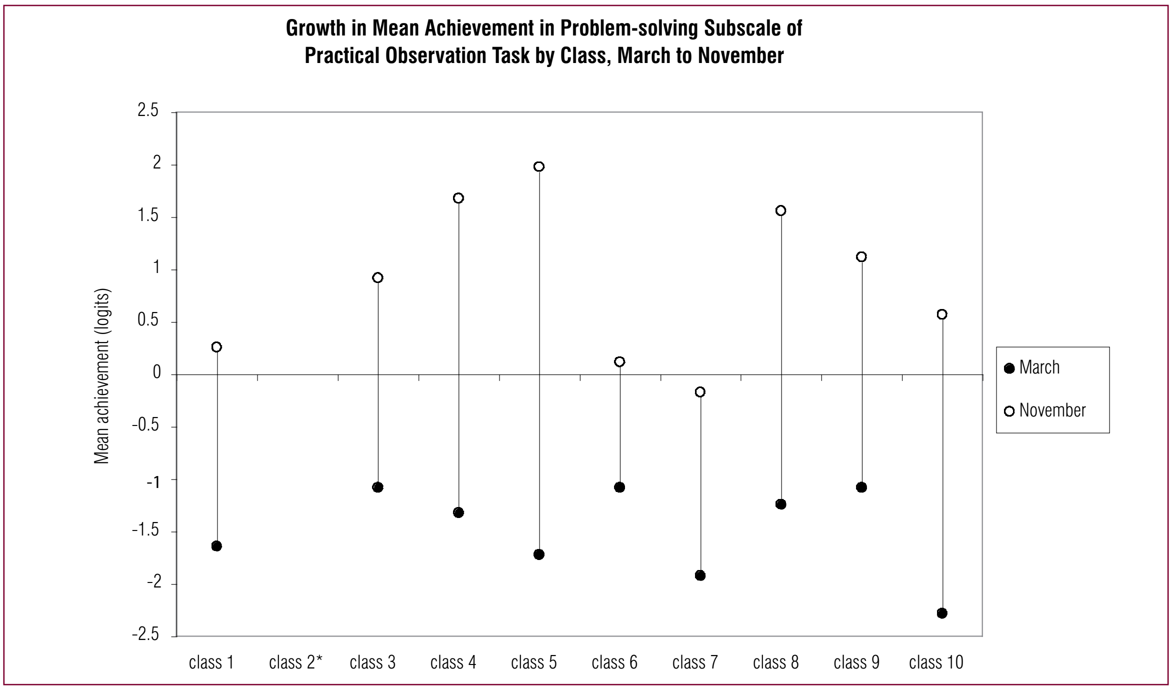
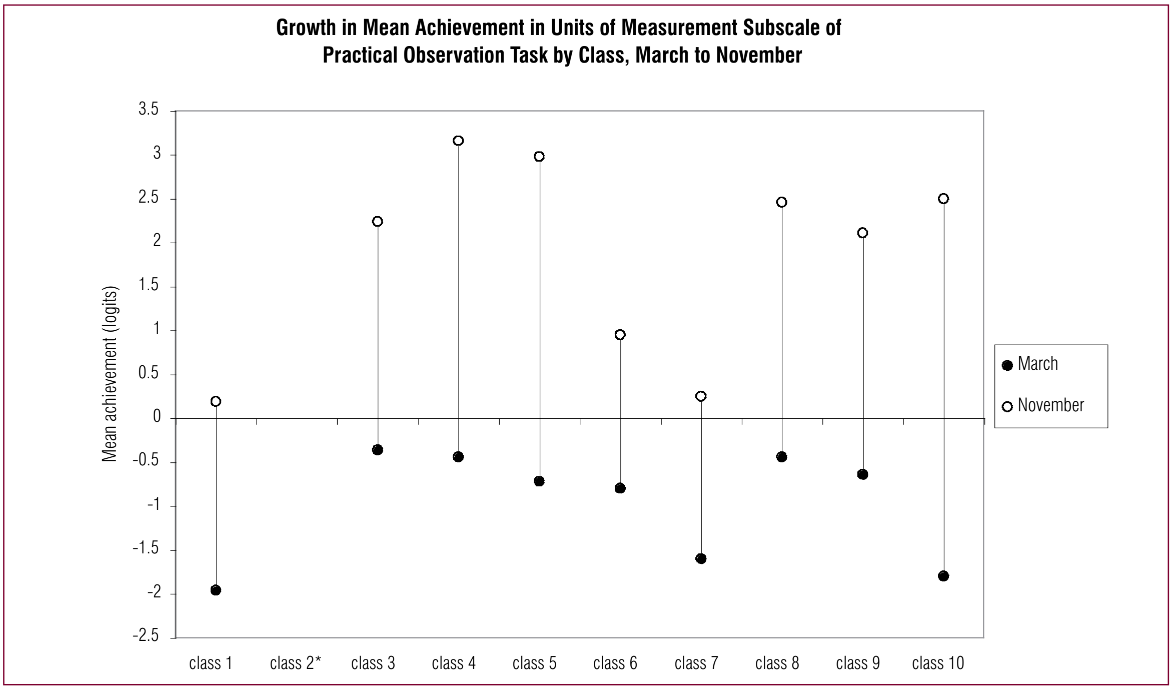


Figure 7. Growth in mean achievement by class for the Units of Measurement subscale of the Task and Observation Schedule, March to November



* Class 2 was withdrawn from the project.

Although there is considerable variability across each of the schools for each of the subscales, there is a clear pattern of general improvement between the March and November observations. In some instances, there has been considerable improvement recorded. The results of the Numeracy Test and its associated subscales cannot be compared directly with the Task and Observation Schedules Subscales. However, both instruments clearly show marked, positive changes in student achievement over the course of the eleven months.

Phase 1: Qualitative Research Methods

The classroom action research was a collaborative undertaking, with the project researchers working alongside the classroom teacher researchers. This collaboration involved fortnightly visits of approximately two hours duration by project researchers to each of the school sites. The purpose of these visits was to conduct shared observations and other data collection (making fields notes, collecting work samples, conducting incidental interviews) during the mathematics class sessions, and to follow this with discussions between the project researcher and teacher researcher.

Collectively, all the teachers involved in the research met for two full days each term. The two project researchers met weekly to share data and observations and to discuss possible interpretive categories, as well as data coding and emerging themes. The two project researchers also met on a regular basis with university research consultants who acted as critical friends to discuss, review and refine the research processes and plausibility of the data analysis and interpretation.

The test results and the students' work samples were analysed in conjunction with the Growth Points (in measurement) and the numeracy Benchmarks and Standards as described in the SACSA framework to provide a formal basis against which students' levels of numeracy could be described. The year culminated in the teacher researchers collating and translating their field notes into a case record focusing on six case students from their class.

Details of each of these components are provided later in this chapter.

Sampling methods

Every effort was made to ensure that the schools and students selected represented the range of abilities and the target groups such as non-English speaking background, low socio-economic status, gender, and indigenous students. This range of sampling decisions reflects the logic of purposeful sampling (Patton, 1980). We believe they best met the constraints of the study in terms of time, resources and manageability. Representative sampling for the quantitative component of the study had to be contained to the selected students from the ten cooperating schools. The ideals of purposeful sampling and progressive focusing in the qualitative component of the study had to be adjusted to ensure a range of target groups was represented in the studies. According to Eisner (1991, p. 205), the goal of qualitative research is 'to refine perception and deepen understanding', while Patton asserts that

> the logic and power of purposeful sampling lies in selecting information-rich cases for study in depth. Information-rich cases are those from which one can learn a great deal about issues of central importance to the purpose of the research, thus the term purposeful sampling". (Patton, 1980, p. 169)

Schools were selected using the Adelaide Social Atlas and school postcodes to determine their socio-economic status and geographic location. The research project began in term one, 2001, with 12 teachers and 280 students from 10 schools. Three country schools were selected with one being selected on the basis that it had indigenous students enrolled. Although twelve teachers began the project, unfortunately, for varying reasons, three of these teachers withdrew. Two of the three teachers were from the country, resulting in the loss of the indigenous students to the project. The third teacher withdrew late in the year, prior to the post practical observation tests. As a consequence of this, the quantitative pre- and post-test data considered only 269 students for the pencil-and-paper test and 235 students for both practical observation tests.

The criteria for selection of schools were that the school had

- a representative range of the target groups of students;
- a supportive leadership team that will support and encourage the teacher;
- a representative range of socio-economic levels; and
- a representative range of geographical locations.

The criteria for selection of teachers for the project were that the teacher

- was prepared to participate in the research and to keep a regular journal and write up case records;
- was a full time classroom teacher in Year 3, 4 or 5;
- had adopted, or was interested in adopting a constructivist basis to the teaching of mathematics/numeracy;
- had a proportion of targeted students in the class;
- was a reflective practitioner; and
- displayed an awareness of current issues in mathematics curricula and numeracy.

The teacher researchers and project researchers collaboratively selected the case students for each class based on the following criteria

- the teacher's knowledge of the students' backgrounds;
- an analysis of the pre-test data; and
- work sample data from the first term's work.

Phase 2: Single and combined case analysis

The second phase of the research design was conducted over a period of 18 months from 2002 to 2003. During this time the 2 project researchers carried out a subsequent analysis of all the action research data collected from each of the 3 selected classrooms, producing 3 single case analyses. These 3 cases were then reanalysed to form a combined case analysis.

The qualitative data included lesson observations (field notes), teachers' case records, students' workbooks, and audio and videotapes of classroom interactions. The method of qualitative data analysis used to systematically analyse these data was informed by Miles and Huberman (1994), Smith (2000), and Freeman (1998).

Throughout this phase the 2 project researchers met on a regular basis with the 2 university consultants to collaboratively reflect, and to review and refine the research process and the interpretations being made. The analysis is further detailed in the research design section below.

Before addressing this detail of the qualitative research components of Phases 1 and 2 in the research design,

some reflections on its theoretical foundations are provided in order to emphasise the distinctiveness and power of this paradigm of educational research.

Researching classroom complexity: The naturalistic paradigm

The intention of this research was to better understand effective mathematics education within classrooms working with a constructivist perspective. Therefore, the theoretical framework for this research was based on principles drawn from a range of constructivist, socio-cultural and cognitive theories (see Chapter 1). Following are the basic tenets that have influenced our analysis of the teaching strategies and possible implications for mathematics education in the primary years.

The teaching of mathematics is viewed as a social interactive experience centred on dialectical relationships between the social and physical setting of the classroom; the facilitation of the mathematical experience; and the students' individual and collective responses to these. According to Wood and Turner-Vorbeck (2001), the desired social and physical setting conducive to learning is one where

- individual knowledge can be challenged and new knowledge constructed through the students' use of language and social interchange;
- knowledge is constructed through experience, using and building onto informal/prior knowledge to make sense of the world;
- both individual and collective meanings are negotiated as students construct mathematical knowledge; and
- metacognition, involving either individual or collective reflection, is a central component of the learning process.

Central to this study of the interrelationship of teaching and students' mathematical learning was the belief that the focus must be on individual students (see Wood & Turner-Vorbeck, 2001), and the collection and analysis of data reflected this belief. A qualitative enquiry within a naturalistic paradigm was selected as the most suitable for constructing meaning about students' learning within the world of lived experience – the social complexity of the classroom. Lincoln and Guba (1985) describe several axioms of naturalistic enquiry and these are closely aligned with the methodology of this research. These are discussed below.

The nature of reality

The naturalistic researcher recognises that there are multiple interpretations of any social phenomenon. The challenge of this axiom is in how researchers can comprehend or interpret social phenomena in relation to the constructed (and culturally influenced) meanings people bring to them. The key is to study the phenomenon in context, in all its complexity, as a whole over time, rather than by fragmenting it (in order to control confounding variables) and apprehending it out of context. Similarly within this research one cannot divorce teaching strategies and student construction of knowledge from the classroom environment when the view of teaching and learning are derived from the constructivist or socio-cognitive philosophies.

Relationship of the 'knower' to the 'known'

Lincoln and Guba's (1985) second axiom of naturalistic inquiry is that the researcher and the researched interact to influence one another; that is, 'knower and known are inseparable'. Qualitative researchers recognise that they are 'human instruments' of data gathering and analysis. An implication for qualitative researchers is the recognition that there is no one true meaning to attribute to social



phenomenon, and the meaning that is constructed will be shaped by interactions between the researcher and the researched. This axiom raises interesting challenges about producing trustworthy (in contrast to valid) research accounts.

In order to make sense of the learning that was occurring, the project researchers needed to become part of the natural classroom setting, and to observe the dialogue between students and between the teacher and students. While the project researchers worked alongside the teacher researchers, they were careful not to be seen as another teacher; they tried not to initiate conversations with the students unless there was a need to make sense of the data they were collecting (and analysing).

The possibility of generalisations

Naturalistic researchers study small samples. Lincoln & Guba (1985, p. 38) postulate that ''the aim of (qualitative) inquiry is to develop an idiographic body of knowledge in the form of working hypotheses that describe the individual case''. But this does not imply that a generalisation from a single case to multiple cases (from one classroom to many classrooms) is not feasible. Wolcott (1995), for example, in discussing the capacity for generalisations within qualitative studies, suggests, ''each case is unique, yet not so unique that we cannot learn from it and apply its lessons more generally (p. 175). Detailed case analyses of 3 different classrooms from different schools in this study captured the diversity of environments and experiences and the similarities. The rich descriptive details of these cases, and their subsequent combined case analyses, rest heavily on the efficacy of generalisations in qualitative research.

> By studying the uniqueness of the particular we come to understand the universal … it is precisely through the encouragement of the case worker in the paradox and living with the tension that creates, holding it open to disbelief and re-examination, that we eventually come to realise the significance of the event, instance or circumstance and the universal understanding it evokes. (Wolcott, 1995, p. 231)

The possibility of causal linkages

There is apparent correspondence between the physical and social settings of the classroom and the quality of mathematical experiences and outcomes enjoyed by the students. Rather than direct causal links, the relationships between these features of a complex classroom are dialectical. Hence the complementarity of Lincoln and Guba's (1985) fourth axiom: 'All entities are in a state of mutual simultaneous shaping so that it is impossible to distinguish cause and effects' (p. 38). We are not aiming to define any simple cause and effect strategies guaranteed to produce advances in students' learning of mathematics. Instead, the research represents an attempt to appreciate the interrelated and multifaceted dimensions of effective mathematics classrooms. Teaching and learning are viewed as social interactive experiences centred on dialectical relationships between the social and physical settings as well as on the mathematical experience and the students' individual or collective response to these.

Values of the researchers

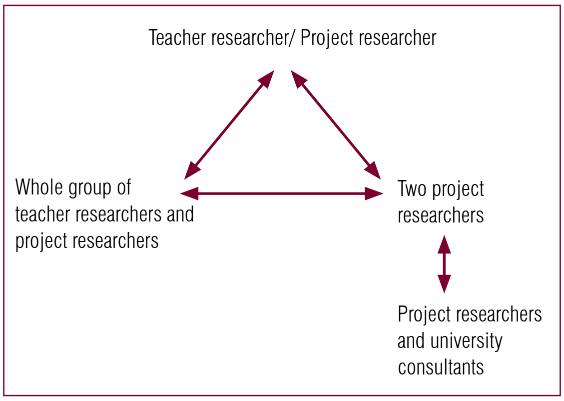
Lincoln and Guba (1985) are unequivocal that naturalistic inquiry is value bound. Values are implicated in the choice of research problem, the choice of fieldwork methods, and the theoretical framework that legitimises the form of interpretive categories. This does not mean that naturalistic studies are flawed, but it does imply that researchers need to be explicit about the values that they are conscious of giving shape to their work. The researchers in this study constantly made decisions relating to what was and what was not relevant data within the bounded system of the case. As Stake (1995) reminds us, qualitative research:

> champions the interaction of researcher and phenomena. Phenomena need accurate description, but even observational interpretation of those phenomena will be shaped by the mood, the experience, the intention of the researcher … It is better to give the reader a good look at the researcher. (Stake, 1995, p. 95)

While this research has been structured to be as objective as possible, the values and theoretical perspectives held by the researchers are still significant and for this reason they are detailed in the report.

The project researchers were aware of the impact of their presence in the classroom and kept this at the forefront of their thinking. They used a range of strategies while in the classroom to minimise the subjectivity. For example, during observations, classroom dialogues were written verbatim. Second, there was a range of discussions and reflections between the project researchers, the teacher researchers, and the university consultants about the observations and interpretations being made throughout the different phases of the research. Figure 8 illustrates the different participants.

Figure 8. Participants involved in the research project



Within each of these reflective discussions the participants became engaged in ‘distancing’ processes. Distancing as used here is an adaptation of Jaworski’s (1996) term to describe how she engaged classroom teachers in reflective analyses of their classrooms. The purpose of this process was to support each of the researchers to stand back and reflect on the work, and to delve more deeply into their own motivations and beliefs than would be possible if done in isolation. As the research evolved and a sense of trust developed between the different researchers, more probing questions, analyses and discussions became evident.

Collaborative fieldwork and classroom action research

Researching in the naturalistic setting of the classroom in Phase 1 of the research encouraged the decision to select an action research design involving the classroom teacher as co-researcher. McKernan, 1996, p. 5) asserts that ‘naturalistic settings are best studied and researched by those participants experiencing the problem’.

Experience gained from working with teachers in previous Catholic Education Numeracy Projects demonstrated to the project researchers that action research encourages teachers



to reflect critically on their rationales (their interpretive/constructed understanding) and adjust their teaching to improve student learning. This research methodology recognises the professionalism of participating teachers, it encourages ownership of the research work, and hence it develops teachers' confidence to share their successful strategies with colleagues. Other teachers reading this work have been motivated to adopt and develop the research findings largely because the findings have arisen from classroom settings where teacher colleagues have been central to the research. This prospect of collegial action research has been acknowledged by McKernan (1996), Elliot (1991), Schön (1991), and Kemmis and McTaggart (1999).

McKernan (1996) illustrates the credibility of action research when he asserts that action research has two essential attributes:

> First, action research is rigorous, systematic inquiry through scientific procedures; and second, participants have critical-reflective ownership of the process and the results. (p. 5)

The research design and methodology in this project is underpinned by contemporary literature on action research. This design employs the action research spiral of Lewin as developed and described by Kemmis and McTaggart (1998) and modified by O'Toole et al. (in prep.). The design involves teachers coming together to develop flexible action plans based on reflections on current experiences. The action research takes place primarily in the holistic setting of the mathematics classroom. Student interactions and learning are observed, and recorded as data by the action-researching teachers in the form of field notes, student work samples and conferencing notes. Data can include the perceived identification of barriers to learning as well as classroom factors contributing to student engagement and success. Participating teachers engage in reflection on their interpretive worlds, both individually through their journals and through group discussion. The reflection phase allows teachers to recognise and stand back from their own experience. This facilitates an ongoing exploration of the nature of the learning taking place within their classrooms and provides them with the opportunity to review their pedagogy and adjust their teaching practices in order to further improve student learning outcomes. The action research design, then, enables teachers to identify effective teaching strategies that support students to improve their numeracy outcomes.

Qualitative research design

The first phase of the project involved a collaborative insider-outsider qualitative inquiry, with classroom teachers and project researchers using an action research model spanning 11 months in 2001. The second phase was the outsider analysis of the data collected and analysed through the action research; this phase spanned a period of 18 months across 2002–2003. Following is the description of the design for each of the two phases.

Phase 1: Collaborative action research

Action research objectives

Henry and Kemmis (1985) describe action research as:

> a form of self-reflective enquiry undertaken by participants in social situations in order to improve the rationality and justice of their own social and educational practices, as well as their understanding of these

practices and situations in which these practices are carried out. (Henry & Kemmis, 1985, p. 1)

Following this definition, action research seeks to generate:

- more profound understandings of teachers' pedagogies as they relate to advancing students' numeracy outcomes
- improvements in the conduct of classroom practices that are integral to these pedagogies
- refinements to the organisational conditions in which these practices are located.

Prior to commencing the action research phase of our project, time was taken to inform the teachers about the research design, especially their role as teacher researchers and the supporting role of the project researchers. The process of the action research began with a reconnaissance – a problem-posing procedure.

Reconnaissance

The initial phase of the action research was a reconnaissance, where the entire group of 10 teacher researchers and 2 project researchers came together to reach a shared understanding of a mathematics education practice that could be improved through self-inquiry and change. It provided an initial analysis of actual classroom situations in the light of shared concern about effective teaching strategies that support all learners to improve their numeracy outcomes within a mathematics classroom based on constructivist/socio-cultural theories.

McNiff et al. (1996) describe the reconnaissance phase as vital 'because action research is about people researching their own practice, and starting from where they are in their real world situation' (p. 29). Grundy (1995) notes that:

> reconnaissance often consists in going backwards and forwards for a while between reflection and collecting evidence. Often the target of action can be generated through discussion (reflection). People sit around and talk about their perceptions of what has been going on and the problems that they perceive. However, very often, what we think is the problem is not the real problem, and collecting some evidence about what actually is going on can help to target the action more acutely. (Grundy, 1995, p. 13)

Grundy further notes:

> although reconnaissance might appear to delay taking action to deal with a problem to improve a situation, it inevitably leads to improvement in understanding, which ultimately might lead to even greater improvement in practice than would otherwise have occurred. (p. 13).

The initial reconnaissance meetings were conducted over two whole days and two afternoons, with further follow-up discussions, data collections and reflections occurring during fortnightly visits by the project researchers. These visits included working in the participating teachers' classrooms after gaining acceptance and establishing a partnership with the teacher researchers. The sessions involved the group researching, reflecting upon and discussing a number of relevant issues prior to refining their

action research focus on ways to improve their pedagogical practices. These issues are summarised below.

1. *Looking within the teacher's unique history and contextual setting*

- The teacher researchers were supported in reviewing how their beliefs and practices have been shaped by their own extensive experiences as mathematics learners and educators. They were assisted in reviewing the students' thinking represented in the quantitative pre-test results. These data provided overall class trends as well as individual students' results, especially those in targeted groups. The teacher researchers were also supported in the selection of case students who were deemed representative of class, gender, cultural background, special needs and thinking levels in their class. The data formed the basis for reflecting on what we do now that is effective teaching; and for whom it is effective. It also invited such questions as: What issues and problems arise in supporting all learners? How best can we describe our beliefs and practices? How does the educational setting within which teachers work influence mathematics education?
- Teacher researchers explored the school context in terms of the educational vision, values and expectations promoted by the school community as a whole, and the role of the teacher within it.

2. *Situating their beliefs in the wider educational community*

Teacher researchers were involved in:

- researching the history of constructivist philosophies along with implications for teaching and learning and then situating their own beliefs and practices within these philosophies
- researching recent literature on the teaching and learning of mathematics
- reflecting on different strategies described as effective in supporting individual students within a classroom environment
- mapping students' thinking by exploring work samples and Growth Points relevant to the measurement attributes being studied. Reflecting on possible effective strategies for assessing and facilitating student learning
- engaging in a series of investigations to deepen understanding of the measurement concepts, and thus providing experiences to reflect on their own educational experiences that formed their ideas on what is effective learning and teaching in mathematics
- viewing education documents such as SACSA framework and Numeracy Benchmarks to reflect on what is viewed as valued and expected mathematical learning by students in the primary years 3, 4 and 5
- reflecting on how this sits with their beliefs, experiences and existing practices.

In the light of this range of insights and reflections, the reconnaissance phase of the action research provided each teacher with a refined focus or question, which would guide their action research. By looking back over their own histories and analysing the context in which they worked, the teacher researchers were able to think ahead about what might be improved and, in doing so, more clearly define the types of changes required. This enabled them to make decisions about the target of action for their research.

Action spirals

By the end of the reconnaissance teacher researchers had refined their thematic concern and formulated their initial plans of action – including selecting the types of evidence that would be collected and the students who would form the basis of the case records. The researchers began their action research in the remainder of Terms 2 and 3 and the first month of Term 4.

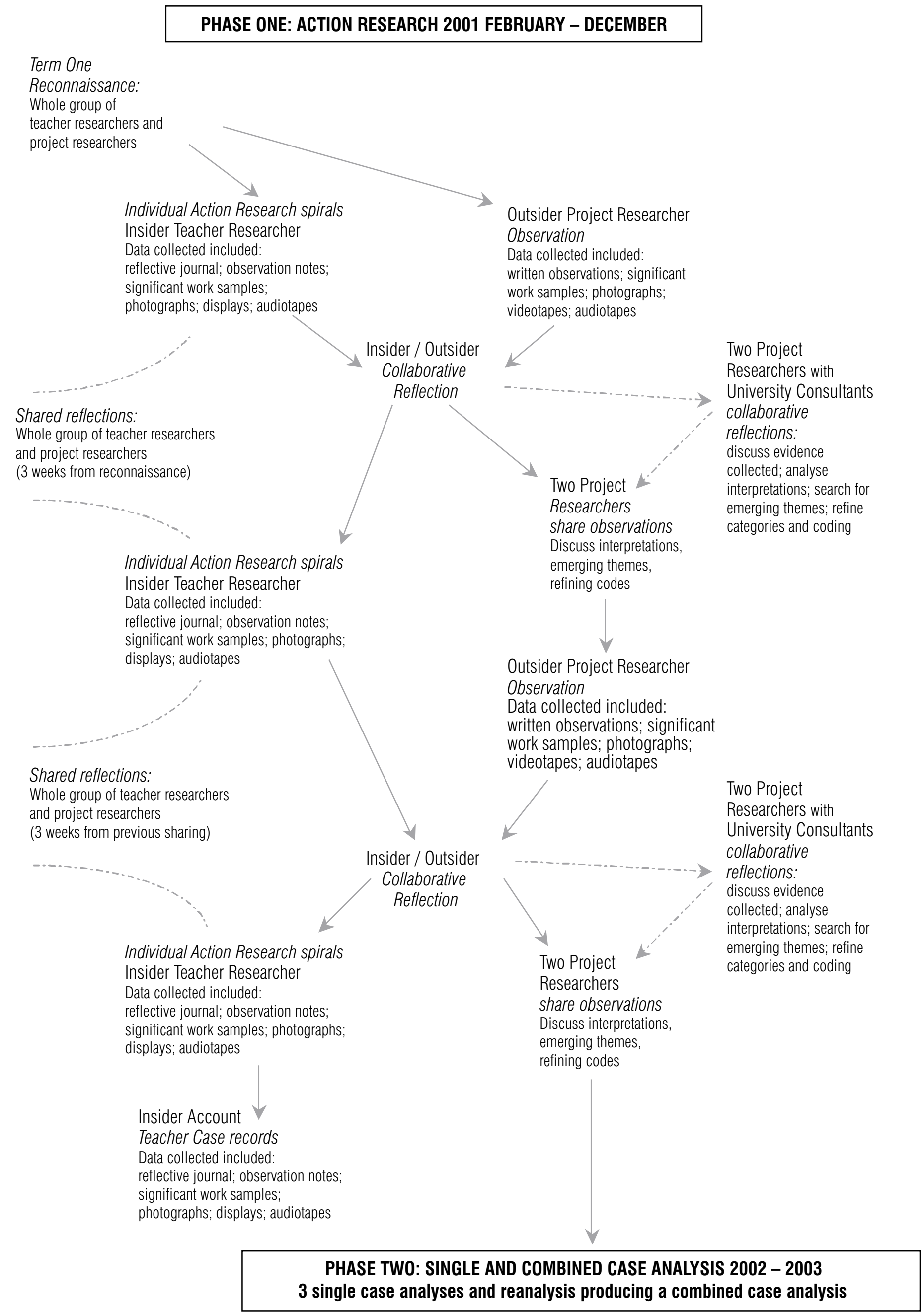
In this research the spiral did not dictate the research pathway. The relationships between the action research moments were seen as iterative rather than cyclical. Kemmis and Wilkinson (1998) emphasise that:

> in reality the process may not be as neat as this spiral of self-contained cycles of planning, acting and observing, and reflecting suggests. The stages overlap, and initial plans quickly become obsolete in the light of learning experience. In reality the process is likely to be more fluid, open and responsive. The criterion of success is not whether participants have followed the steps faithfully, but whether they have a strong and authentic sense of development and evolution in their practices, their understanding of their practices, and the situations in which they practice. (Kemmis & Wilkinson, 1998, p. 21)

Figure 9 illustrates how the action research spirals occurred, and shows the relationship between the individual teacher researchers and the project researchers. The left side represents the whole research group coming together for shared discussions and reconnaissance activities. The central section represents the actions of the teacher researchers; the right side represents the actions of the project researchers, and the arrows indicate the interactions between the two categories of researchers.



Figure 9. Relationship between the individual teacher researchers and the project researchers during the action research phase



Insider role: teacher researchers

After the reconnaissance and the setting of the initial action research plan, the teacher researchers (insiders) independently implemented their plan of action in their classrooms, made their observations and reflected on their practice. The number of spirals within the action research model completed by the teacher researcher between the reconnaissance stage and the insider/outsider observations and reflections, varied from teacher to teacher. The action phase takes its shape from the planning phase. However, action was not always controlled or restricted by the planning moment. While teacher researchers worked with their plan, if some initial evidence indicated that a teaching strategy needed to be changed to support a particular student's learning, they were able to make this change immediately.Grundy (1995) comments:

> Action in the real world of social practice is not just a matter of implementing a set of carefully worked out plans. We need to be more flexible than that. Professional practice requires judgement and ability to think and act 'on our feet'. Thus we will not be slaves to the plans. (p. 14)

Outsider role: project researcher as participant observer

Once the action research commenced, a project researcher (outsider) visited the schools of each teacher researcher fortnightly. The initial purpose of these visits was to be a participant observer within the class to support the teacher in the collection of evidence. The types of data collected as evidence included written observations, video recordings, audiotapes, and the selection of significant work samples.

Insider/outsider collaborative reflection

The second purpose of the visits was to arrange for insider/outsider collaborative reflections. These sessions (approximately 1 hour) were conducted immediately after the observation sessions. These reflections helped provide part of the triangulation process whereby the project researcher and the teacher researcher discussed and worked through two sets of data sources and two mindsets (comprising different but complementary interpretive categories related to mathematics education). The collaboration was to come to some shared understanding about what had occurred and to gain insights into the teacher's thinking behind their strategies.

The two researchers shared their initial observations, each taking turn to provide their perspective (applying their interpretive categories) to what happened in the teaching session. The discussions were centred on sharing field notes and analysing teaching strategies and individual student thinking. When views were dissimilar, the participants took the time to negotiate meanings.

Congruent/similar interpretations

If the teacher researcher and the project researcher views were similar and agreed upon, the discussion moved on to what the evidence was indicating about the effectiveness of the plan. One of 3 outcomes would follow. The evidence would:

- affirm the plan, and enable the plan to continue with a refined focus of the data collection to gain deeper insights
- affirm the effectiveness of the plan and enable data collection to continue
- indicate a need to change the plan resulting in classroom changes.

Dissimilar interpretations

If the interpretations of teacher researcher and the project researcher differed then discussion between the researchers, including exploratory and clarifying questions, led to deeper analyses. Once a shared understanding was reached then reflective discussions moved on to what the evidence was indicating about the effectiveness of the practice. This again resulted in one of 3 outcomes. The evidence would:

- affirm the plan and enable the plan to continue with a refined focus of the data collection to gain deeper insights
- affirm the effectiveness of the plan and enabled data collection to continue
- indicate a need to change the practices resulting in classroom change.

During the reflection moment of the action research cycle, the project researchers used questions aimed at supporting the teacher researcher to distance themselves in order to become more reflective and critical. As well as this, the teacher researchers' explanations and questions also supported the project researchers to rethink their interpretations. This provided the project researcher with a deeper insight into the teacher researcher's thinking, individual students' thinking, and greater awareness of the effect of the teaching strategies.

Project researchers' collaborative reflections on observations

The two project researchers met weekly to discuss evidence collected from each school, and to analyse and interpret the evidence in order to identify emerging themes. These themes then formed the basis of the interpretive categories and coding schemes for analysis. These sessions also informed the issues and design of the whole group teacher researcher sessions, which occurred twice a term.

Project researchers' collaborative reflections with university consultants

The two project researchers met periodically with university consultants throughout both research phases. The purpose was to discuss data from each school, to analyse interpretations, and search for emerging themes, and to refine the interpretive categories and coding. During this reflection the university consultants supported the project researchers in distancing themselves from their data, and in evaluating the effectiveness of analytical interpretations and processes.

Whole group shared reflections

These sessions involved all the teacher researchers coming together with the project researchers to share their research data, and to reflect collaboratively on their teaching strategies and the effect of these on student thinking. The sessions allotted time to discuss evidence collected from each school and to analyse interpretations made, as well as search for any common emerging themes from across the different sites.

Insider account: teacher case records

Data collection by the teacher researchers included field notes, written observations, reflective journal records, photographs of students at work, and photocopies of significant work samples. Several teachers also produced audiotapes of conversations between themselves and the students, while a few teachers kept or photographed their displays as evidence for their case records. The case records were collated as the year progressed, but in Term 4 the teacher researchers spent considerable time reflecting on the year's data in the light of their teaching and learning. The focus, while maintaining the whole class perspective, was primarily on the 6 case students. Likewise, the evidence collected was also mainly from the case students.

Phase 2: Single and combined case analysis

The purpose of this phase of the project was to conduct a subsequent analysis of the data collected during the action research fieldwork from 3 selected classrooms. First, data from each of the 3 teachers were reanalysed in depth and documented as a single case. Then all 3 cases were reanalysed to produce a combined case analysis. The qualitative data included lesson observations, teachers' case records, students' books, audio-and videotapes that were continuously and systematically analysed using methods informed by Miles and Huberman (1994) and Smith (2000). This analysis was aimed at probing the question:

> What do effective teachers do to construct classroom environments that engage all children in thinking and working mathematically?

Effective teachers were determined by the students' performance in the pre- and post-test in conjunction with the initial analysis of the qualitative data from these classes. The qualitative data provided the most information and rich examples of teaching strategies that supported students to improve their numeracy outcomes. *Engagement* is viewed as the student persisting in working on a task to the point of making sense of the mathematics. *Classroom environment* represents classroom norms, climate, resources, and activities, physical space and interactions.

Throughout this phase, the 2 project researchers met on a regular basis with the 2 university consultants to reflect collaboratively on the data and the interpretations made to ensure reliability and validity of the findings.

In Phase 1 as the data was collected the analytic categories of the 3 single case analyses and the combined case analysis were initially aligned with 3 interrelated theoretical constructs: social setting, physical setting, and mathematical experience, as discussed in Chapter 1 and earlier in this chapter .

These categories were gradually developed into a series of codes and questions, which informed the data collection and analysis. Once more data was collected and detailed it was reanalysed and the categories were further refined. The matrix of codes and categories, as per Miles and Huberman (1994), was the basis for our interpretations.

Phase 2 involved another, more-detailed process of analysis that was undertaken involving the use of partitioning categories and codes for the case analyses, work samples and observations. When codes and categories were cross referenced it became clear that the initial codes needed to be modified. This resulted from the process of 'dissembling and reassembling' (see Freeman, 1998). Because of this process it was possible to make more profound interpretive statements about the teaching strategies and possible implications for mathematical learning in the primary years.

Selection of cases for core detailed case analysis

Due to the time constraints on this study it was not possible to analyse the volume of data collected from all 10 teachers, so the selection criteria were informed by the quantitative and qualitative data as detailed in the overview. The overall results from both pencil-and-paper and practical contextual tasks indicated that significant growth had occurred across all classrooms, particularly in the case of Language Background Other than English (LBOTE) students. We therefore decided to select the 3 teachers whose classes showed greatest growth consistently across all the measures as the subjects of the detailed case analysis.

As shown in Figure 10, Class 10 clearly demonstrated the greatest increase – approximately 1.8 logits of growth – followed by Classes 1, 4, 5 and 8, which all showed an increase in excess of 1.2 logits.



Figure 10. Growth in mean achievement from March to November for the pencil-and-paper numeracy tests for the 10 project classes

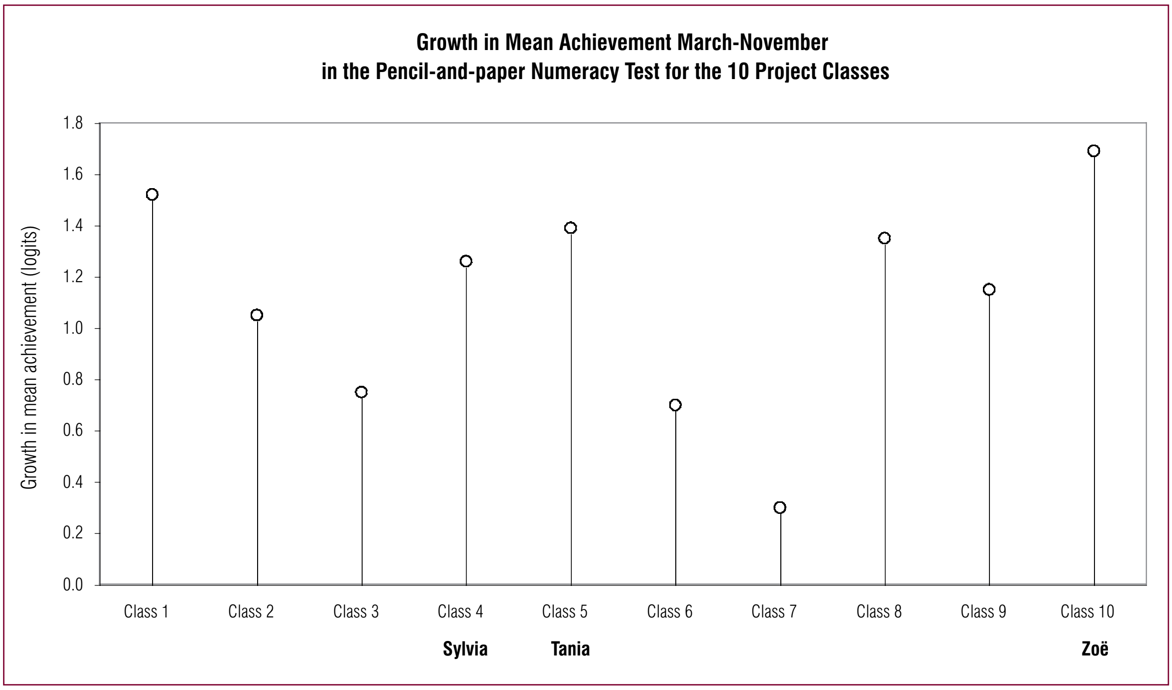
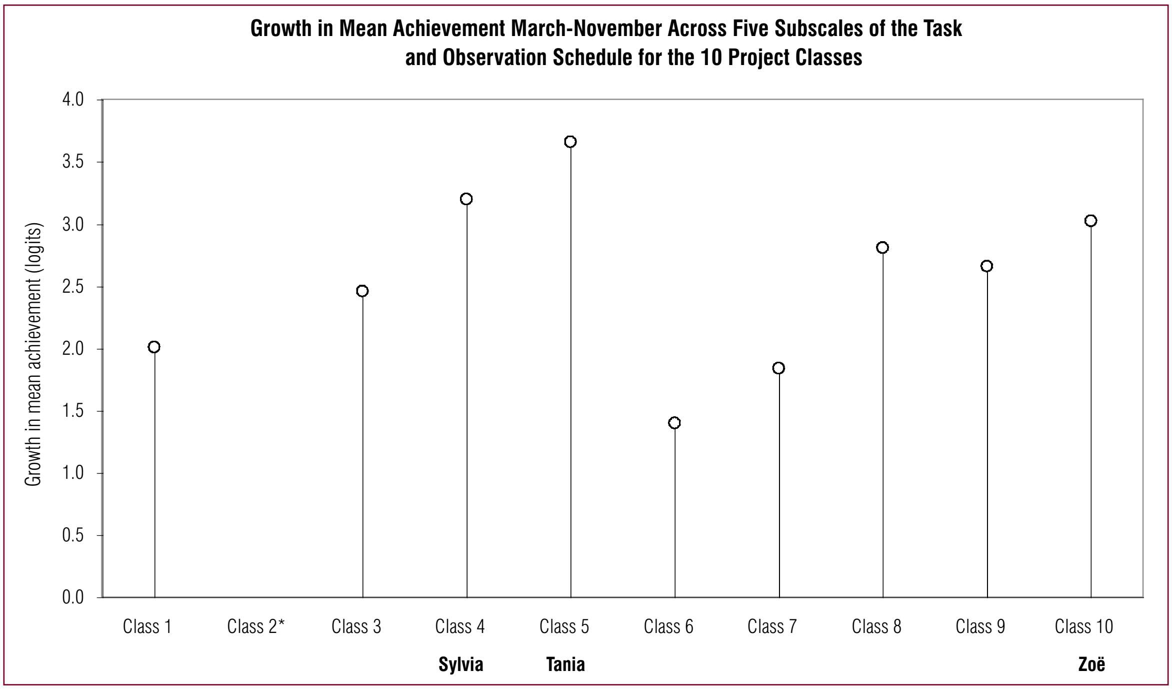


Figure 11 shows the overall results for the 5 subscales of the practical observation test. Classes 4, 5 and 10 demonstrated greater growth than the other classes. Therefore the decision was made to select Classes 4, 5 and 10 for the more detailed analysis. It is important to note that Class 2 was withdrawn from the project just prior to the post practical test and therefore there is no growth data available for this class.

Figure 11. Growth in mean achievement from March to November for the 5 subscales of the practical observation tests for the 10 project classes. Note that Classes 1 and 8



* Class 2 was withdrawn from the project.

The classes selected were Zoë's Year 3 class (Class 10), Tania's Year 4/5 class (Class 5), and Sylvia's Year 5/6 class (Class 4). Two of these classes contained LBOTE students – 4 in Zoë's class and 10 (4 Year 4 and 6 Year 5) in Tania's class. Also, special needs students were represented in 2 out of the 3 classes – 2 in Sylvia's class and 1 in Zoë's class – thus providing us with our targeted students across the range of year levels 3, 4, and 5.

Due to the constraints on this research, the teacher researchers were not able to be involved in the single or combined analysis phase of the project. The 2 project researchers worked together to systematically analyse the data collected from Phase 1 of the research. This analysis was directed toward identifying any illuminating ideas (Lincoln & Guba, 1985) and their probable implications for mathematics learning in the primary years (Wolcott, 1995; Eisner, 1991; Simon, 1996).

Chapters 3 and 4 describe respectively the single and combined case analyses. The documentation related to Phase 1 of the research is consolidated and summarised in the teacher case records. These are not part of this report but have been central to the analysis for the 3 single case analyses and combined case analysis.

Case Analyses

Introduction

To investigate effective teaching that assists all students to optimise numeracy outcomes it is imperative that the researchers get to where the action is – the classroom. The classroom is a socially dynamic and complex environment. When teaching is viewed from constructivist or socio-constructivist philosophies, one cannot separate teaching strategies and student constructions of mathematical understanding from the classroom environment. Classroom research is very constrained, however, in what it can say – it cannot adequately reveal or represent the natural dynamics of classroom life. The case analysis approach best embraces and scrutinises these complexities.

The case analyses that follow portray the classrooms of 3 teachers – Tania, Zoë and Sylvia. As described in Chapter 2, the classrooms were selected because between them they included a breadth of year levels (3 to 5) and significant numbers of LBOTE and special needs students. Table 5 shows the numbers of students, including the numbers of LBOTE students in each class, and the increase in mean numeracy score from March (pre-test) to November (post-test). Although the three classes were not comparable in terms of size, numbers of LBOTE students, or mean numeracy scores, the increases in mean numeracy scores in each case were high compared with most of the other seven classes in the research project. These three classes were also selected because they represented a spread of year levels.

Table 5. Numbers of students and mean numeracy scores (logits) for the 3 case study classes

	Class		
	Zoë	Tania	Sylvia
Year level	3	4/5	5/6
Number of students in class	29	29	30 (14 in Year 5)
Number of special needs students	1	0	2
Number of LBOTE students	4	10 (4 Year 4 6 Year 5)	0
Mean numeracy score: March (logits)	-1.40	0.31	-0.20
Mean numeracy score: November (logits)	0.29	1.57	1.19
Increase in mean numeracy score (logits)	1.69	1.26	1.39

Project researchers used audiotapes and videotapes, classroom observation field notes, teachers' case records and students' work samples to construct an analysis. These case analyses demonstrate that there are no simple cause-effect strategies that produce advances in mathematical thinking and understandings. They also show that to appreciate the multi-faceted dimensions of effective mathematics classroom it is necessary to analyse and describe the complexities of the classroom. Through the descriptions and interpretations of the data we will show how the physical and social settings and the mathematical experience come together to assist all students to think mathematically.



Central to researching the mathematics classroom is the interrelationship of teaching and students' mathematics learning within the socio-cultural learning environment. The main theme running through the case analyses is that the development of mathematical understanding improves through a socio-cultural environment (Vygotsky, 1978; Bruner, 1990; Ernest, 1996). The social context provides the students with opportunities to discuss, share and acquire knowledge, to display their own ability, to learn to respect other people's ideas and to investigate new skills. Within this social environment students work together as they construct their own, as well as a shared, understanding of mathematical concepts.

In this project, the research focused on the following:

- the physical setting of the classroom
- the social setting in the classroom
- facilitation of the mathematical experience
- the teachers' use of the Growth Points as a conceptual framework when planning, facilitating and assessing the mathematical experience.

The physical and social environment and the mathematical experience the teachers created in their classrooms supported individual students to build and develop their mathematical understanding. The settings were interrelated and complemented each other in creating a positive learning environment.

Physical setting

The physical setting of the classroom is defined by the classroom layout, the mathematics resources available to the students and the displays, for example, samples of students' work.

Social setting

The social setting encompasses the aspects of the classroom that come together to create an environment where students can interact with each other as well as with the teacher:

- grouping strategies
- protocols – encouraged behaviours in the classroom
- students' working styles and possible implications for teaching and learning
- classroom interactions.

Grouping strategies

In each of the 3 classrooms the students worked in small groups designated by the teacher. Students did not necessarily remain in the same group for different investigations, and LBOTE and special needs students were supported by other students in their groups.

Protocols: encouraged behaviours in the classroom

Similar protocols were established in each of the 3 classrooms. If students required assistance they generally sought help from another member of their group first before asking the teacher.

Students' working styles

The social setting of each of the classrooms, including the grouping strategies and protocols, the tasks assigned to the students and the support offered by each teacher, led to two styles of working – collaborative and co-independent. These two styles shared a number of features. In both styles students supported each other, were involved in discussions, and recorded their own thinking. In the co-independent style, however, the students worked individually and called

on other members of the group for support. The styles used by the students are detailed below.

Collaborative style

Students worked collaboratively in small groups, or in pairs, to investigate mathematical ideas and to solve problems. In this style of working each member contributed to a group effort to discover solutions and construct knowledge (see Cobb, 1992; Edwards & Mercer, 1987). The students discussed the task, the strategies and the tools they chose to use throughout the investigation, resulting in shared findings. Students were assessed individually throughout the learning process.

Co-independent style

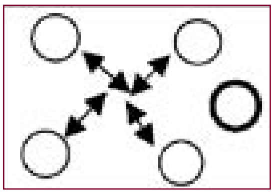
A co-independent style was evident when students worked on their own task using their own strategies alongside others who might be doing the same investigation. The students chose their own strategies and applied the mathematical ideas they wished to use in their investigation. Students were encouraged to seek support from other members of the group even when at times members of a group might be working on different investigations or be at different stages of the same investigation. Consequently they sometimes shared ideas and strategies but they recorded their own thinking using their own language. As a result, the work samples of students sitting next to each could be quite different. In a co-independent style of working, students were assessed individually through conferences and from their work samples.

Classroom interactions

Systematic analysis and interpretations of the observational data from the 3 classrooms resulted in the identification of a number of different modes of interactions between the teacher and students and amongst the students themselves. These interactions evolved or were orchestrated by the teachers as the students engaged with the mathematical activities in the classroom and supported the students as they constructed meaning and built deeper understandings of mathematical concepts. Classroom interactions provided the teachers with opportunities to find out about students' understandings and prior knowledge of the concept in a non-threatening environment. The teachers not only used teacher–student interactions to support students as they constructed their own understandings but also encouraged student–student interactions as a means of supporting each other in building shared understanding.

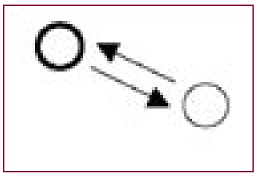
The interactions that emerged from the observations have been classified into 6 categories, although not all of these categories were observed in all 3 classrooms:

Category 1 Small group interaction with or without the teacher observing



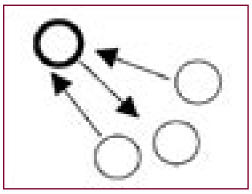
Students interacted with others, discussing strategies, making decisions and debating conflicting ideas. This type of interaction occurred as the students worked either collaboratively or co-independently, and it was not teacher initiated. The teacher was not invited into the discussion even when she came close by to observe.

Category 2 One-to-one interaction (conference) between a student and the teacher



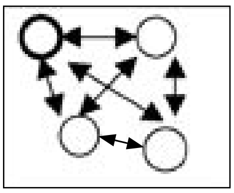
During a conference, the student remained seated within the group while other group members continued with their work. A conference always began with the teacher asking the student to talk about what they were doing and the strategies being used in order to draw out the student's understanding of the task being undertaken.

Category 3 Small group interaction (conference) with the teacher directing the group



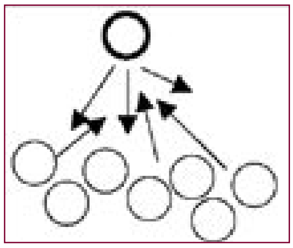
Sometimes the teacher conducted a conference with a group of students. In this case all members of the group took part in the discussion.

Category 4 Small group with the teacher interacting and observing at different times



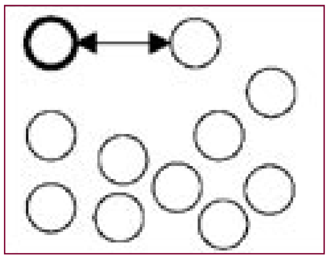
At times the teacher interacted with a group of students, but would often stand aside and observe, perhaps rejoining the discussion at a later time.

Category 5 Whole-class interaction with the teacher



These whole class interactions typically occurred at the beginning or at the end of a lesson.

Category 6 One-to-one public conference in a whole-class setting



Occasionally, at the end of a lesson, a conference would take place between the teacher and one student, with the rest of the class as audience.

Each of these categories is elaborated in the case studies using extracts from recorded conversations gained during classroom observations.

The mathematical experience

Conferencing

Conference is a term used in other areas of education, namely literacy education, and is used here to describe a structured conversation between the teacher and a student. Working within a constructivist environment the teachers engage the students, either individually or in small groups, in reflective conversations (conferencing) as their understanding in mathematics is challenged and scaffolded. Since the teacher and student generally conference one-to-one, the teacher's questions will depend on the students' responses and their prior knowledge. As a result of conferencing students may go about their investigations in different ways. The conference allows for individual students' thinking to be recognised, valued, refined, extended and challenged within a non-threatening environment. Through conferencing and recording, students verbalise their thinking and understanding, making their understanding accessible to their peers and to the teacher. The teacher uses a series of questions to support the students in linking their prior/informal knowledge to the new knowledge.

Growth Points

One objective of this research was to determine whether Growth Points, if used as a conceptual framework, could be useful to teachers in helping individual students within a classroom environment to build mathematical understandings. The teachers in this research used Growth Points when planning, designing tasks, and supporting and assessing students' thinking. O'Toole et al. (1999) researched and documented Growth Points associated with a range of classroom activities (R–10). Their work built on Pengelly and Rankin's (1985) study on Growth Points in linear measurement by further exploring the unifying ideas as well as other attributes in measurement.

The teacher researchers were introduced to the Growth Points in the reconnaissance phase of the research. Through the use of students' work samples and classroom observations, the teachers were provided with a holistic view of measurement rather than a list in isolation to students' thinking. Although all participating teachers used the Growth Points, the case analyses demonstrate that the teachers put their own interpretation on them. The nature of Growth Points is that they are not finite and teachers were encouraged to add to them.

During the course of the analysis, however, it became apparent that there was a possibility that Growth Points were being seen as a set of outcomes for all students to pass through rather than as a description of the different pathways students went along as they constructed their mathematical understandings. The name Growth Points therefore could be misleading.

These insights led us to extend the concept of Growth Points in the case analyses to what we eventually renamed as a web of *Possible Learning Connections*. This development is discussed in Chapter 5 'Implications for Mathematics Teaching and Numeracy Outcomes' and in Appendix B.



Case Study 1: Tania

At the time of this study Tania had been teaching for 18 years and had been in her current school, St Hilary's School, for 8 years. Tania is one of several key mathematics teachers who have been actively researching the implications of constructivist philosophy for the teaching and learning of mathematics.

School Setting

St Hilary's School is a Reception to Year 7 Adelaide School located in an area identified as low-socio-economic (Adelaide Social Atlas, 2002). The school community is culturally diverse, with students from 20 different cultures, including Vietnam (the largest group), El Salvador, Poland, Macedonia, Hungary and The Philippines. Of the 262 students in 2002, 45 per cent had school cards (the measure commonly accepted in South Australian schools as an indicator of socio-economic disadvantage) and 35 per cent were identified and funded as students who are Language Background Other than English (LBOTE).

The school has a history of extensive in-service literacy and numeracy training, with a strong focus on LBOTE perspectives. An LBOTE support teacher is employed for two days per week to work alongside class teachers. In 1999, Catholic Education South Australia and St Hilary's began a project to provide professional development for all the teaching staff at the school in numeracy/mathematics education. The project ran for three years. During this time two teachers emerged as key numeracy teachers, one teaching in Early Years and the other, Tania, in the Primary/Middle Years section of the school. Working in conjunction with a numeracy consultant from Catholic Education South Australia, these two key teachers were given time out of their classrooms to continue the in-service program for the school staff.

Alongside the school in-service program the two key teachers were also involved for three years in a complementary action research project outside their school. This project involved other key teachers from Catholic schools across the state.

Physical setting of Tania's classroom

There were 29 students in Tania's Year 4/5 class, 10 of whom were LBOTE students (4 Year 4 and 6 Year 5 students). The physical setting of Tania's classroom was designed to allow students to interact as they carried out their investigations. Three main aspects – the layout, resources and display – were significant in assisting students as they constructed and made sense of mathematical concepts.

Layout

Tania's classroom was set up for paired and small group work, with the tables spaced to allow the teacher easy access to all students. When investigating, students sat either at their desks or on the floor at the front of the class. Sometimes, when they needed more space, students worked on the veranda outside their classroom, within sight of the teacher.

Resources

Sufficient quantities of a range of materials, which included non-standard and standard units/tools, were placed on the students' tables at the beginning of the sessions.

Displays

Displays comprised a variety of student work samples representative of the range of thinking from the mathematics sessions. The purpose of the displays, according to Tania, was to value individual student thinking and to

support the students in sharing and linking their informal ideas and language with conventional mathematical language. Although the student work samples included both informal and conventional language, the captions written by the teacher provided visual models of conventional terminology.

Social setting of Tania's classroom

Grouping strategies

All the research observations showed students working in mixed ability groups within year level groupings selected by Tania. When selecting groups Tania took into account the 10 LBOTE students and ensured that each group had at least one English-speaking student who could model the language and support the LBOTE students. All students were allowed to select their own seating arrangements within the groups.

Protocols – encouraged behaviours in the classroom

Protocols operating in the room were designed to support and encourage students to work collaboratively/co-independently. All students were expected to engage with the tasks and to share, discuss and debate ideas and strategies. They included the following rule:

If experiencing difficulties try asking 3 peers. If still unsure, indicate that you need help and go on with something you can do until the teacher visits you.

(Tania's case record observations)

Throughout the learning process students were expected to verbalise and document their thinking and understanding using their own language as well as the conventional language of mathematics.

Classroom interactions

Data were collected from Tania's classroom fortnightly – a total of nine observations. The observations indicated that Tania spent most of the lesson time interacting with students either individually or in small groups. The physical set up of the room and the protocols that were in place facilitated this level of interaction, as students were free to discuss their understandings with each other and with the teacher. The students were not inhibited by the teacher's presence when she came to sit next to them to listen and sometimes join in their discussion.

The interactions we observed in Tania's classroom will be further analysed and categorised to determine whether the patterns of interactions had any implications for students' mathematical thinking and understanding. (see 'The mathematical experience' below).

Students' working styles

In all of the observed sessions the students were seated in groups and were involved in paired or small group interactions, in which they discussed mathematical ideas and strategies. In some sessions, the students worked together on a single task whereas in others they were involved in individualised investigations. This observation led us to the decision to analyse the students' working styles in Tania's classroom. In the 9 sessions observed the students adopted 2 working styles – collaborative and co-independent.

Collaborative style

The students in Tania's class discussed and worked together as they investigated and solved problems but they were assessed individually. In Excerpt 1 the students were working collaboratively.



Excerpt 1

One girl and 2 boys were working as a group, investigating the conservation of area. Together they were discussing the problem and the strategies they could use to solve it.

Sam	*Is the relationship between these shapes the same?*
Julia	*Yes, they all make the same shape.*
Andrew:	*How can we measure the area?*
Sam	*With centimetre blocks.*
Sam	*[Sam moved the shapes around] I think checking to see if they do relate would be good.*
Andrew	*We have measured the inside of the shape to find out the relationship between the big triangle and the two smaller triangles and the parallelogram.*
Sam	*How do we know which triangle?*
Julia	*We'll call it the small triangle and the large triangle.*
Sam	*How are we going to record it?*
Andrew and Julia	*You have to say they have the same area.*

The students discussed and shared strategies for finding the areas of the shapes and their relationships in order to solve the problem.

Co-independent style

The students shared ideas and strategies but continued to make their own decisions about how to investigate and how to record their own thinking using their own language and understandings. Evidence of this style of working was found in observations of the students, in the students' workbook recordings and in Tania's case record, as shown, for example, in Excerpts 2, 3 and 4.

These excerpts and the work samples shown in Figures 12 to 15 demonstrate that even when a group of students work together, their subsequent recordings differ according to each student's interpretation.

Excerpt 2

Julia, Nina, Lynne and Sam were working together. The task was to explore the relative area of the pieces in a 7-piece tangram. They discussed and shared ideas but carried out and recorded the investigation in their own way. They were very focused on the task – looking at the shapes, turning them around, visually exploring the size and discussing their observations and strategies for validating their conjectures.

Nina	*That's the biggest shape [pointing to the triangle].*
Lynne	*Two of the small triangles has the same area as the square.*
Sam	*How will we record that, have you recorded that?*
Nina	*Can we draw them?*
Lynne	*You can draw the shapes if you like. That's what I'm going to do.*
Nina	*I'm drawing shapes that fit into each other and I'm going to say they have the same area.*
Lynne	*I am going to write this is the biggest shape.*
Lynne	*We have to let them know then how we decided that this shape was the same size.*
Nina	*Yeah, we measure it.*

In Excerpt 3 Sam and Julia provide feedback to each other's conjectures and work towards conclusions about the relative areas of different shapes. Again, their recording reflects their independent thinking while working together.

Excerpt 3

Julia — *Now, don't we have to make the same shapes? You can make a square [she begins the next task].*

Sam — *[looks up from writing] So what's the next question?*

Julia — *Do these two shapes go together?*

Sam — *Yeah, they do.*

Julia — *Hey, continue on to the other one – the two smaller triangles take up the most area also.*

Sam — *No they can't take up the most area.*

Julia — *No … the two smaller triangles take up the least area.*

Sam — *Ah but have you tried it with the other shapes – they're not exactly the same.*

Julia — *Yes they are.*

Sam — *They're not. I can see from here they're different sizes.*

Julia — *Well you're wrong.*

Sam — *You're right, the two smaller triangles take up the least amount of area.*

Julia — *Let's write it.*

Sam — *Now let's write something about these ones.*

The group then discusses the best way to record their ideas.

Sam — … colour the shapes so you know which shape you are talking about. All the shapes beside the big triangle … no, it doesn't make sense.

Julia — All the shapes besides two big triangles make exactly the same size. Half a square is the same as half the triangle.

Sam — Two triangles make two halves of the square?

Julia — Because if you lay these on here they match the area. [Julia lays the triangles on the middle-sized triangle, then tries to help Sam link his idea to his work in the last lesson, involving the tangram and slicing and rearranging rectangles].See, how you've done here. [points to his recording from that session] Just put it together.

Sam — Yeah, that orange thing … I just thought of it.

Figure 12. Work Sample: Julia

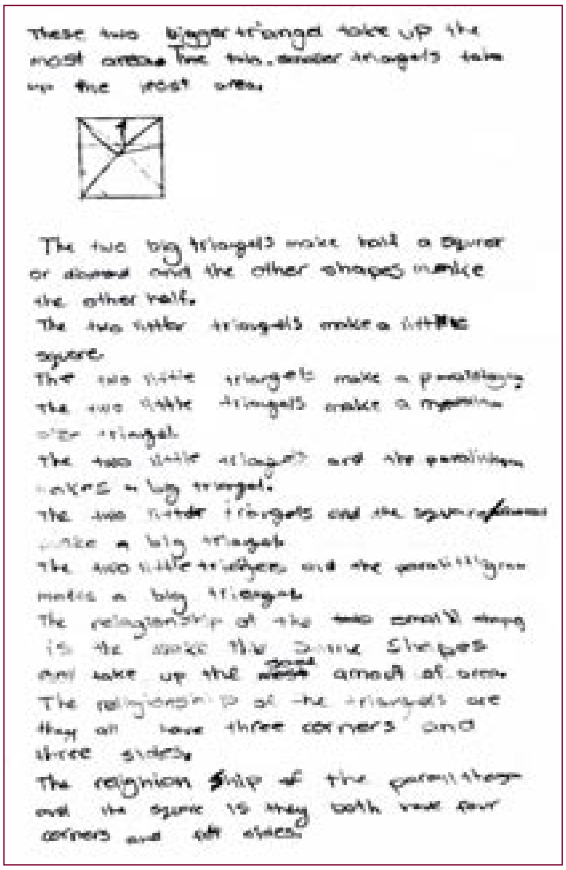
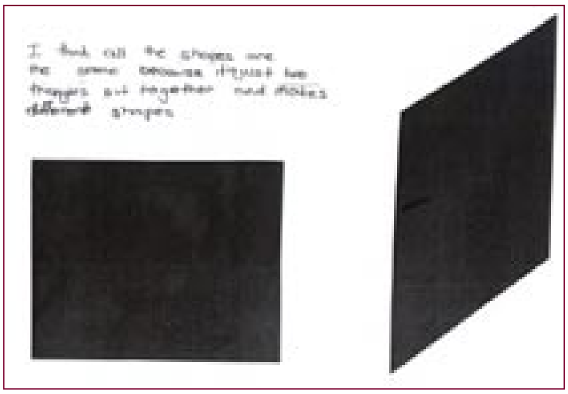


Figure 13. Work sample: Sam



In Excerpt 4, Lynne and Nina arrive at different strategies for determining the area of a shape, although interacting as they work co-independently. Their work samples follow in Figures 14 and 15.

Excerpt 4

Lynne and Nina separated from the others. They had two different ways of verifying why the triangle and the square have the areas.

Lynne could see that by overlaying the triangle on the square, if she cut off the two corners that overlapped and placed them on the square gaps, they would match.

Nina *That's because if you cut the triangle down the centre and rearrange it on the square it would be a direct match.*

They each chose to write down their own ideas. Tania suggested to them in the next session that they might like to compare why their two strategies both verified their idea.

Figure 14. Work sample: Lynne

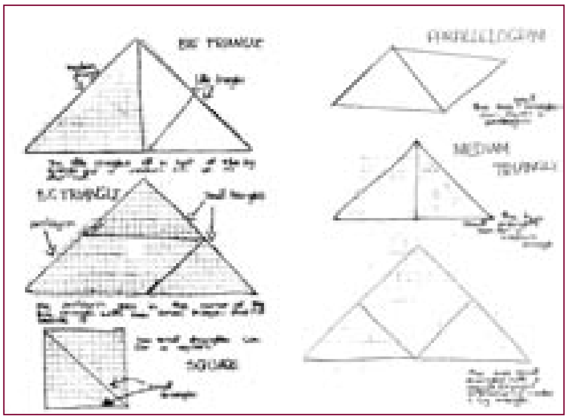
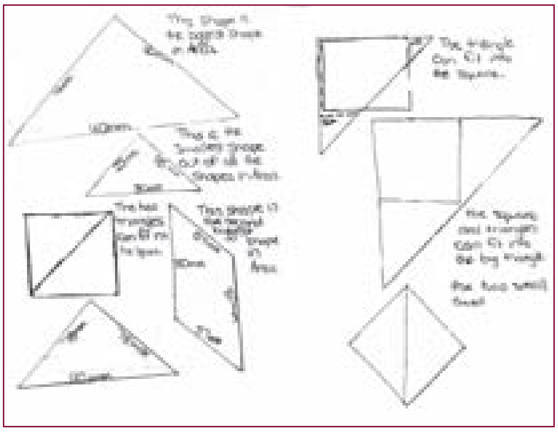


Figure 15. Work sample: Nina



In both the collaborative and co-independent styles of working students interacted as they were investigating. In the co-independent style, however, students could choose to work individually and ask for assistance whenever it was required, or, if they were all doing the same task, they could choose to discuss and share ideas and then record their own thinking. The physical and the social setting that Tania put into place supported the emergence of each of the working styles. The mathematical experience and the nature of the tasks also contributed to these two styles, as discussed below.

The mathematical experience

This section will portray how Tania facilitated the mathematical experience for all students in her room. First, we analyse the classroom interactions, and then the questions and responses during these interactions and their possible consequences on the construction of mathematics/numeracy knowledge. We will address Tania's use of Growth Points in her planning as part of the facilitation of the mathematical experience.

At the beginning of each mathematics/numeracy session Tania briefly informed the class of what they were expected to do. The major features of the lessons observed were individual or small group interactions (conferencing). The following discussion is therefore an analysis of students' learning in relation to these conferences.

Student investigation time and conferencing

In the sessions observed Tania spent most of her time conferencing, averaging 4–5 conferences in a typical lesson. Lessons were typically 60 minutes, with each conference lasting 10–15 minutes. The data showed that Tania initiated 90 per cent of the conferences, with the students initiating the remaining 10 per cent.

In a teacher-initiated conference, Tania observed students investigating and listened to their conversations before she started questioning them. Tania's first question usually elicited the student's current reasoning or understanding related to the investigation. This was the pattern for most new conferences. During a conference Tania sometimes left a student or small group if she felt they needed time to reflect on their thinking or on the strategies they were using, or when they needed time to link prior knowledge to the mathematical ideas being investigated. Excerpt 5 illustrates the different sorts of questions Tania used when conferencing.

Excerpt 5

Tania observed Sam, who was sitting with another boy. He had selected a square, an isosceles triangle and a rectangle and he had ordered them according to their perimeter.

Tania	Sam, what have you done here?
Sam	Ordered by perimeter.
Tania	Why did you use millimetres for your measures?
Sam	Because if I want to be as accurate as possible I need to use as small a unit as possible and that is a millimetre.
Tania	You have not labelled your recording so why does 59 x 4 give you the perimeter?
Sam	Each side is the same.
Tania	Same what?
Sam	Length, so I measured one and timesed by 4.
Tania	I see, and here why 85 x 2?
Sam	These two sides are the same length so I measured one and doubled it then added. I did the other same for the rectangle; I measured each of the different length sides and x 2 then added.

Sam pointed to the height and base as he described his findings.

Tania left Sam to continue his investigation. When she returned, Sam had had begun to compare by collecting data on the area of the shapes. Tania noted that he was not including the parts.

Tania	So I see you are counting the whole squares inside the triangle.
Sam	Yes.
Tania	Why not the parts?
Sam	Are they necessary?
Tania	So do they need to be counted as a measure of the area?
Sam	Yes, I'll cut them out and slice them up to make whole squares.



Sam's response to the question: *What have you done here?* was brief, prompting Tania to ask the next question about millimetres. These two questions gave her an insight into Sam's reasoning. Further on in the conference, when Tania realised that Sam had not measured the area accurately, she used scaffolding techniques to help him understand that the parts were necessary and could be measured. Tania then left Sam to recount the squares.

Once Tania had assessed and affirmed a student's thinking, she sometimes allowed the student to continue with the task without further questioning. At other times she proceeded to challenge students to reflect on their mathematical ideas or strategies and to find out the conventional mathematical language that was applicable to the problem. Excerpt 6 illustrates this strategy.

Excerpt 6

The students decided to break into small groups to measure and calculate the areas of the fishponds so that they could compare their answers to work out which one occupied the greatest area. Lynne suggested that it was essential that they all used base10 blocks to calculate the area. When James questioned this suggestion Lynne responded.

Lynne	*How else can we compare our answers? Every fishpond must be measured the same way.*

James suggested it would be easier to use flip blocks to calculate the area. Lynne argued that it would take forever because the blocks were so small and also were not really a unit. Lynne responded when asked by Nina to clarify this.

Lynne	*Flip blocks are not really a unit that you measure in. For example, you do not hear people saying this shape has an area of 3 flip blocks whereas unit blocks can be converted to numbers like 1 long equals 10 units.*

Once they had resolved their differences, the students set to work. Lynne and James measured their pond using the base10 100s, longs (10 cm bar) from the MAB blocks and 2 cm blocks. The students covered the fishpond shape with blocks. However, when they calculated the area of the fishpond, they wrote that the area was 422. They did not understand that different units were being used and that they must convert their measurements to the same unit. At this point Tania intervened to try to assist their thinking.

Tania	*Which fishpond looks the biggest?*
Lynne	*Ours is.*
Tania	*But group one's fishpond is smaller than yours and they've measured it as 3475 blocks. How can this be?*
James	*They've made a mistake.*
Lynne	*No, that's probably right if they only used centimetre blocks. There would be a lot of them.*
Tania	*Have they only used centimetre blocks?*
Lynne	*No! They probably changed them to unit blocks. That's why their answer is so big.*
Tania	*Do you need to change your 100s,10s, and 2 centimetre blocks to unit blocks?*
Lynne	*Yes, otherwise you can't add them together and there is no way that we will be able to verify that our fishpond has the greatest area.*
James	*Can we just change the 100s to10s?*
Lynne	*Yes! Then you have to change the10 blocks to units so that you can count them altogether. Otherwise we will never be able to verify which pond has the greatest area.*
Tania	*Do all the fishpond areas need to be converted to unit blocks?*
Lynne	*Yes, otherwise we can't compare areas.*

(Tania's case record)

When Tania challenged the students her questions encouraged them to verbalise their thinking, which she continually assessed. The students' responses determined the next question or action and whether it was another challenge or one that called for the learner to make connections to their prior knowledge.

If students responded positively to being challenged, Tania further challenged them, supporting them in building onto their existing knowledge. For example, Tania's question: *Do all the fishpond areas need to be converted to unit blocks?* required the student to rethink their use of units when measuring area. If the student needed support Tania asked questions that scaffolded the students to reflect on their prior knowledge and build new understanding.

In a few instances, when the students did not have the required prior knowledge, Tania asked a series of questions described as *funnelling* (see Bauersfeld, 1980). These questions are somewhat closed but seemed to produce the desired outcome by directing the students' thinking. This suggests that Tania was continually challenging students to link their prior knowledge to the mathematical understanding, creating a network of ideas onto which to build mathematical understanding.

Purposeful questioning by the teacher assisted students to reflect on their thinking. As the students assimilated the sorts of questions Tania asked, they began to ask similar questions of each other and possibly of themselves. It could be hypothesised that such practice may encourage students to reflect on and use their mathematical knowledge in problem solving. In Excerpt 7, for example, after collecting and analysing her data, Lynne shared her findings with the rest of the group. It seems that she understood the concept of area of a rectangle and she was able to call on the relevant information to explain how area could be calculated.

Excerpt 7

Lynne collected data on 15 rectangles, looking for a relationship between the sides and the area. Lynne was heard sharing her findings with other members of her group.

Lynne *There is no need to individually count the squares; all that is needed is to times the length and the width.*

When asked how she worked it out, Lynne elaborated on her thinking, demonstrating that she had been able to generalise to obtain a formula that worked.

Lynne *Because you can make rows and columns, remember we did this in other lessons.*

Lynne stressed however, that her formula could only be used with rectangles and squares 'as they have equal sides'. She was asked to clarify this.

Lynne Rectangles allow there to be equal groups. Whereas, if the formula is used on a triangle there are so many parts left over and when counting the squares on the grid paper they do not match.

(Tania's case record)

Lynne's ability to identify relationships and to link this relationship with other pieces of information quickly and efficiently to solve the problem suggests that, at least in this particular instance, she was showing some characteristics of metacognitive thinking.

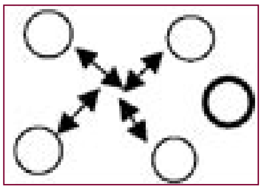
When Tania was satisfied with a student's current level of thinking she concluded the conference, allowing the student to continue with the investigation. At other times, when students had difficulty in making a link between ideas Tania moved away to give them time to reflect, but later returned to these students and continued the conference. In small group conferences Tania challenged the students' thinking then stepped back while the students' debated and discussed their ideas and strategies. Tania re-entered the

conversation if she felt the students had lost direction in the conversation or if she found that the students required an additional challenge. It appeared Tania left when the discussion was becoming focused in the direction that was more supportive of their investigation so the students could think through their understanding as a group.

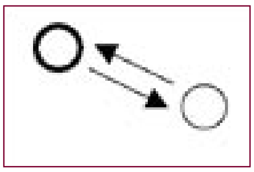
Interpretive summary of classroom interactions

Four patterns of interactions that supported the students as they constructed meaning and mathematical understanding were evident during the observations of Tania's classroom:

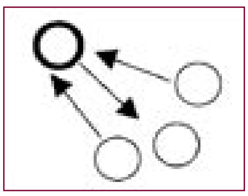
Category 1 Small group interaction with or without the teacher observing



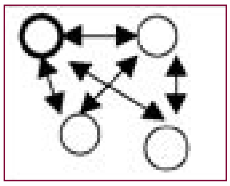
Category 2 One-to-one interaction (conference) between a student and the teacher



Category 3 Small group interaction (conference) with the teacher directing the group



Category 4 Small group with the teacher interacting and observing at different times



These interactions evolved or were orchestrated by Tania as the students engaged in mathematical activities in the classroom. These different types of interactions are illustrated using excerpts from the case study data.

Category 1 Small group interaction without the teacher observing

Excerpt 8

A group of 5 students were collecting data on rectangles and discussing their thoughts. After Nina had collected data on 15 rectangles, she informed the rest of the group that there was an easy way to find the area of rectangles.

Nina *You times the base and height together.*

Tuan asked Nina to justify her answer. She explained to Tuan that if he counted the number of flip blocks he would obtain an area of 18. Then if he multiplied the height by the base he would also get 18, thus verifying her answer. Rachael interrupted and emphasised that it was not the height that was being multiplied but in fact the length. Nina corrected Rachael and informed her that it did not really matter as it depended on where you were sitting as to how the orientation of the shape appeared.

Tuan *Could we then call the height, the width?'*

Nina *Yes! As they both mean the same thing.*

Rachael informed Nina that she could see two groups of nine, while Tuan asserts that he could see nine groups of two. When Tania approached the children, they were trying to understand why the formula worked. Rachael and Tuan were advising Nina that the groups must be involved in some way. James, who had already discovered the relationship, interrupted to explain that it has to do with breaking the rectangle into rows and columns.

Tuan	*Yes! But is that the same as breaking them into groups?*
James	*Yes it is the same. You are breaking the rectangle into groups just like you are breaking it in rows and columns.*

(Tania's case record)

This type of interaction indicates that the students within Tania's class had taken learning as a social activity where their role was to support and challenge each other to think through issues, to force each other's thinking and to reflect on their decisions together.

Category 2 One-to-one interaction (conference) between a student and the teacher

In the research observations, 40 per cent of the interactions observed were one-to-one, with 90 per cent of these interactions initiated by Tania, and only 10 per cent by the students. This type of interaction is illustrated in Excerpt 9, where Tania assessed and responded to Sam's thinking as she assisted him to work from and extend his own understanding.

Excerpt 9

Tania observed Sam, who was sitting with another boy. He had selected a square, an isosceles triangle and a rectangle and he had ordered them according to their perimeter.

Tania	*Sam, what have you done here?*
Sam	*Ordered by perimeter.*
Tania	*Why did you use millimetres for your measures?*
Sam	*Because if I want to be as accurate as possible I need to use as small a unit as possible and that is a millimetre.*
Tania	*You have not labelled your recording so why does 59 x 4 give you the perimeter?*
Sam	*Each side is the same.*
Tania	*Same what?*
Sam	*Length, so I measured one and timesed by 4.*
Tania	*I see, and here why 85 x 2?*
Sam	*These 2sides are the same length so I measured 1 and doubled it then added. I did the other same for the rectangle; I measured each of the different length sides and x 2 then added.*

Sam pointed to the height and base as he described his findings.

(Tania's case record)

Tania's questions created opportunities for students to verbalise their understandings. Having to put their thoughts into words encouraged them to reflect on their thinking. The one-to-one interaction was also a strategy used by Tania to continually assess individual students' understanding and provide the students with immediate feedback. By regularly spending time with individual students Tania implicitly communicated to the students that they were valued as learners.

Category 3 Small group interaction (conference) with the teacher directing the group

Tania sometimes worked with a small group of students, questioning the group as a whole, with different members of the group responding. Tania initiated 90 per cent of these interactions while the students initiated the rest. During this interaction Tania ask open-ended questions of the group, inviting any member of the group to respond. Excerpt 10 illustrates a small group conference where Tania directed the discussion.

Excerpt 10

Six girls were working out the area of the footpath. The students placed the metre square on the path and were counting how many times they did this. They were not taking into account the part of the square metre that was extending beyond the path. Tania intervened:

Tania — *Let's see what is happening here.*

Tania asked the students to place the metre square on the pathway and guided them to realise that part of the metre square was hanging over the edge of the path.

Tania — *Are you covering all the space?*

Student — *Some is going off?*

Student — *And some here. [The metre square was protruding over both sides.]*

Tania — *How much is hanging?*

Trinh — *It is about 10 cm.*

Tania — *Are we talking about linear measurement or are we talking about area measure?*

Felicity — *So what about this part on the other side? [Referring to the part that was hanging over the edge]*

The students moved the metre square around.

Tania — *We're losing a bit here.*

They then re-measured the path, but Trinh, who was taking the lead, kept ignoring the parts.

Felicity — *We'll fold it in half and then there will be the centimetres.*

Trinh — *60 cm.*

Tania — *60 cm?*

Felicity — *No, it is not very accurate.*

Tania knelt down at the same level as the students.

Tania — *Come on now … let's think about this. What is a metre square now if you fold it in half like you suggested?*

Felicity — *You will get 50 cm.*

Tania — *Will it be 50 cm? What do the others think?*

Student — *[Stepping onto the metre square] This is 90 cm … this is more than half.*

Student — *What about … 90 cm is too high.*

Tania — *Why not fold this out and see?*

Student — *Fold out the metre square.*

Student — *Half.*

There were now two students doing this.

Tania — *Now, when you fold it that way what is the measurement? What do you have?*

Trinh — *You have a rectangle.*

Tania — *Yes, you have a rectangle but you had 1 square metre to start with, what is this part in square metres?*

Felicity — *It is 50 cm.*

Tania — *What is that in square metres?*

Trinh — *It is like a rectangle … it is not a square any more so it cannot be a square metre.*

Tania — *What is it?*

Trinh — *A half.*

Tania — *Where is the whole that you started with?*

Student — *There.*

Tania	*What is the whole?*
Felicity	*A square metre.*
Tania	*Now what do you have?*
Trinh and Felicity	*Half.*
Teacher	*Of what?*
Trinh	*Half of a square metre.*
Tania	*You have half of square metre … it is not 50 cm, because we are not dealing with a metre … we are dealing with a square metre therefore this is half of a square metre.*

Initially, the students did not fully understand that none of the metre square should hang over, and that it is important to ensure that this does not happen if they were to get an accurate measurement. Through the conference with the teacher they were supported to think through the mathematics within a contextualised setting.

Small group interactions (conferences), such as that seen in Excerpt 10, provided a social forum in which the students were called upon to verbalise their thinking. Tania was able to assess, respond and build onto individual student thinking within this social setting. Having to verbalise and discuss one's thinking in a supportive small group served the same purpose for the individual as Category 1 interactions. Within a small group environment, however, this type of interaction encouraged students to draw and build common understandings and language.

Within this type of interaction Tania controlled and monitored the group dialogue/discussion, ensuring that the direction and common understanding was useful and supportive of the students' learning. In this small group environment Tania provided the students with immediate feedback as she continually assessed individual's understanding as well as the group's understanding. Category 3 interactions also gave Tania the opportunity to model reflective questions to the students. She attempted to provide a social environment that supported the development of metacognitive skills by encouraging the students to ask reflective questions of themselves and others.

Category 4 Small group with the teacher interacting and observing at different times

Tania sometimes worked with a small group of students as they were investigating. The interactions were between the teacher and all the students in the group. At times she stepped out of the conversation and observed as students discussed and debated their ideas and strategies. Tania re-entered the conversation when the students appeared to lose direction, or if she found the students required an additional challenge or support. She left the students to continue as a group when the discussion became more focused in the anticipated direction. Excerpt 11 illustrates this pattern of interaction:

Excerpt 11

Nhi	*Look at this, look at this shape.*
Trang	*Yeah, that one has too many jagged edges.*
Tania	*If you think that do you want to have a go at this one over here, because you don't have to work on the same one.*

Tania stood back and observed:

Nina	*[In the background] That is going to be hard, I'll do this one.*
Joanne:	*How do we do this? Don't we measure this one out there and then we measure this one here? [She was referring to part of the pond with 3 straight edges of a rectangle]*
Nina	*You can use a ruler.*
Joanne	*We could measure the perimeter.*
Trinh	*No, we're not, we could measure that [width] and that [line segment] and then times it by that and then we will find the area inside.*

Tania intervened:



Tania *Do you want to explain what you were going to say to them? Why will that give you the area?*

Tania again stood back and observed.

Nina *Because. [Nina was shy and reluctant to explain].*

Joanne *Because that and that is equal to that and that [length and width of the rectangle] … which is part of the pond.*

Trinh: *Because these 2 are equal and these 2 are equal so you can times the height and the base.*

Nina *But there is no height up here.*

Trinh *Yes, but we can draw it … this is dividing the pool up.*

Nina *So we are doing up to here, OK. [Nina was happy with this – she had been unsure why they wanted to use a formula when the shape was not a rectangle].*

Tania intervened again.

Tania *When you times the height and the base why will that give you the area?*

Nina shook her head.

Tania *Does anyone know?*

Trinh *It is because this side is equal to that side.*

Tania *Do you want to explain what you mean when you say they are all equal sides and why this means you can times the height and the base to measure the area?*

Trinh *Because if we break it into two different shapes, that is a rectangle and the top side is the same length as the bottom side and the two sides are equal. When these sides are equal (pointing to matching sides) then the shape can be divided equally into rows and columns (drawing the columns and rows across the rectangle section as she speaks). The length of the base is the number of columns and the length of the height gives you how many in each column because it tells you how many rows, so instead of covering the whole space with blocks we can just measure that and that with the ruler and times them together.*

Tania *So Nina do you understand why Trinh wants to use this strategy?*

Nina *I think so, now I see the rectangle, but what about the other part that's not a rectangle?*

Trinh *Then we can use the cm^2 blocks for the other part of the pond and add them together*

Nina seems to understand this so Tania leaves the group.

Within this type of interaction students were called upon to verbalise thinking, understanding and strategies, and to build a common understanding. In Category 4 interaction Tania did not control the group dialogue/discussion, the students had opportunities to discuss and challenge each other's thinking as she stood back and observed. Tania moved back and redirected the conversation when she felt that some students appeared to be lost. Her questions directed the students to clarify their thinking and understanding of the concepts behind the mathematical process they were suggesting or using. By allowing students to share the control of the group discussion Tania implicitly conveyed the message that students could support each other and that she respected their knowledge.

Table 6 summarises the types of interactions in which Tania engaged during the observed lessons. In 60 per cent of Tania's interactions, she worked with small groups of students. In 20 per cent of these interactions students interrelated amongst themselves – discussing, debating, making decisions and reflecting on their selection of ideas and strategies, while the teacher observed, or occasionally stepped into the conversation to challenge or redirect the students.

Table 6. Summary of Tania's interactions

Category of interaction	Percentage of interactions
Category 2 One-to-one interaction (conference) between a student and the teacher	40%
Category 3 Small group interaction (conference) with the teacher directing the group	40%
Category 4 Small group with the teacher interacting and observing at different times	20%

Although 60 per cent of the interactions emphasised the importance placed on social interactive learning within a small group, it is significant that 40 per cent were individual interactions between Tania and a student. Small group interactions also gave Tania access to individual students' thinking. We conjecture that by valuing and monitoring growth of individual students, Tania ensured that individual learning needs were catered for.

Pivotal to these interactions taking place and contributing to students' learning was the collaborative environment that Tania created by the implementation of class protocols and supportive grouping strategies. The protocols in place supported both collaborative and co-independent work where students were expected to share responsibility for their own learning and that of others, to have accessibility to the teacher as required but to respect each student's right to uninterrupted time with the teacher. The strategy of mixed ability and mixed year level groups facilitated acceptance of the protocols. Tania ensured that within each group someone could read, understand and interpret the investigation/task and support discussion as well as acting as the more capable peer/s who could support other group members.

The social setting in Tania's class encouraged the students to interact as they constructed and made meaning of mathematics. The main theme that emerged from the patterns of interactions was the engagement of students in a range of social interactions as individuals or as members of a group. The focus was to provide opportunities for the students to verbalise and reflect on their thinking and choice of strategies, as they built deeper mathematical understanding.

Analysing the types of questions Tania used during a conference

Tania's extensive use of questioning when conferencing students was one of the main features of her teaching strategies. In this section, we analyse Tania's questions to see if there were any patterns in the effect on student learning. This more detailed analysis uncovered two different types of questions, which seem to result in students either:

- reflecting on their thinking and explaining current strategies
- reflecting on and explaining their current reasoning.

Reflecting on students' thinking and explaining current strategies

The reflective questions that occurred throughout the conferences appeared to achieve their goal of encouraging the students to reflect on their thinking and data. Tania often employed these sorts of questions when she wanted the students to rethink by reflecting on their data. Excerpt 12 illustrates these reflective questions.



Excerpt 12

Josh was measuring the area of a fishpond. He had covered the space with blocks, denoting that he was aware of the concept of area. But because he used 2centimetre cubes and MAB blocks to measure area the teacher wanted to find out his thinking.

Tania *Good, how do you write that? [pointing to the 66, 2-centimetre cubes].*

Josh *Is this cm cube?*

Josh was unsure, writing down 66 and then erasing it.

Tania *No, no, I think there are 66. There are 66 of this sort [2-centimetre cubes]. My question is, How much space do they take up? They are 66 what?*

Josh took a ruler and measured the sides of the cube.

Josh *2 cm.*

Tania *Are you sure?*

Josh was again unsure. He took four 1-centimetre cubes (he was familiar with the 1-centimetre cubes.) and made a block with an area of 4 cm^2. Although he was using cubes, he was focusing on the faces that he had used to work out the area, hence his next response:

Josh *It's 4 cm^2 (this is the area of a face of a 2-centimetre cube).*

Tania *It's got an area of 4, is it right, now you have 66 of them, what's its area?*

Josh *66 times four.*

Tania *[comparing the method used by another group] Can I say the area is 21 hundreds, 66 cubes 88 long ones. Why is yours different from theirs? What did they do?*

Tania's question*: 'Are you sure?'* aimed to refocus Josh's thinking and encourage him to reflect on the data and what he was trying to find out. Tania's question prompted him to check his measuring tools and to reassess his findings. This forced him to reflect on the relationships between the units. Tania questioning placed Josh's idea in conflict. Tania then further challenged Josh by asking him to compare his thinking and findings with those of the other students, encouraging him to reflect on relationships between the units.

When Tania was observing a small group of students who were constructing rectangles from flip blocks and then collecting data on the area and the linear lengths of the rectangles, she asked them if they had found a quick way of calculating the area of the rectangles. One student, speaking for the group, informed Tania that there was a pattern: 'you can times one length by the other length and that will give you the area'. Tania responded:

So does that work for all of them? Check and see if it works for all.

(From observation 5)

Tania's question was encouraging these students to reflect on the idea that they had developed from the analysis of their existing data and to then see if this idea would work for other situations. This challenge question supported the students in developing a sense of the mathematical process. It affirmed their pattern search but it also challenged them to take the next step to see if it worked for new situations.

Reflecting on and explaining current reasoning

The most frequently used questions were those that focused on encouraging students to explain their thinking. It seems that sometimes Tania was aware of a student's reasoning but wanted the student to think through his or her own reasoning. At other times Tania attempted to uncover the student's existing ideas and reasoning, ready to challenge and build onto it.

The following examples illustrate questions that Tania asked for these purposes.

Observation 1	*So why does 59 x 4 give you the perimeter?*
Observation 2	*So why do you think you add instead of multiply?*
Observation 3	*What do you mean add a zero? …*
	What do you do to go from 12 cm to 120 mm?
Observation 5	*So you said timesing … what does that mean?*
	Why do you times by 4?
Observation 7	*How do you know that?*
	How are you going to know how much space it occupies?
	Why does it all have to be in units?

These questions called upon students to explain their reasoning behind the choices of ideas and the strategies they used when working through problems and drawing conclusions. Tania used this form of questioning to provide opportunities for students to verbalise and think aloud, thereby forcing them to reflect on their own thinking. This often resulted in the students evaluating the decisions they had made and at times realising that they had made an inappropriate decision, that they had incomplete data, or their conclusion or reasoning about their data was incomplete.

The classroom observations showed that Tania focused her conferencing on establishing each student's current understanding and knowledge as she attempted to bridge the gap between what the students already knew and what she intended them to learn. By conferencing students individually, Tania became aware of each student's mathematical knowledge and understanding and how they were making sense of the mathematical experience. Her case record and the classroom observation data illustrate that students were asked different questions though they may all have been working on the same task. The questions had the same purpose, which was to develop certain mathematical ideas, but Tania followed different pathways, depending on the students' responses.

Using the Growth Points

All the teachers who took part in this study made use of the Growth Points (see Appendix B), some to a lesser extent than others. Because Tania had worked previously with Catholic Education South Australia consultants she had some experience with these sets of Growth Points. At the beginning of this study Tania worked with the project researchers and the other teacher researchers in using the Growth Points to analyse the pre-test data of the students in her classroom. The information gained from this analysis provided a basis for Tania to assess the range of students' thinking in her class. This analysis informed the planning of units of work on measurement, and the selection and design of a series of possible investigations, with the aim of further supporting students' development in linear measurement and area. As confirmed by classroom observations and the excerpts, Tania also used the Growth Points to inform her continual assessment of the students' thinking.



Planning the framework for a unit of work

Before starting the unit on measurement, Tania devised a teaching plan that we will refer to as her planning framework. This planning framework provided a structure for Tania and helped her to think through the mathematical ideas she wanted the students to attain and the possible pathways individual students could take as they explored the mathematics. The framework also guided her with possible questions she could ask to support the construction of mathematical concepts and further challenge the students' thinking. Included in the same planning framework are the mathematical investigations/tasks (referred to as starting points), the conventional mathematical terminologies the students may need to know and use, and the materials that could support the students in their exploration of mathematical ideas. When designing a planning framework Tania considered the following:

- mathematical ideas
- materials
- terminology
- the range of possible investigations/task (starting points)
- possible teaching questions.

Another feature, which was only briefly mentioned in the case record, was the involvement of parents and the community in enriching the mathematical experience for the students, initiated by Tania. The students surveyed their parents about how they used linear measurement and area knowledge in their work or leisure activities. The parents and the members of the local community were invited to come in as guest speakers to share their experience and use of mathematics in their life. This focus was influenced by the Growth Points, which described the importance of the students recognising and appreciating how linear measurement is applied and developed within our local and wider community (O'Toole et al., in press,).

When Tania used the Growth Points document to assist her planning, she referred to the Growth Points as mathematical ideas. She also added her own interpretations though she kept to the essence of the descriptors. Her planning framework demonstrates that she selected only the Growth Points she considered relevant for her students. The mathematical ideas (see Figure 16) for the linear measurement unit, for example, were described in terms of Growth Points, showing that she used the term interchangeably with the *mathematical ideas*. From the list of Growth Points Tania created two shorter lists, which she labelled as short-term focus. Growth Points are not to be used chronologically, although Tania documented them sequentially, signalling that she may have considered or used them in this way.

Mathematical ideas

Figure 16. Mathematical ideas for the linear measurement unit

a. Developing an awareness of length as an attribute that can be measured
b. Classifying, comparing, matching and ordering linear measurement through direct comparisons
c. Distinguishing between different aspects of linear measurement such as length, height and perimeter
d. Choosing and using non-standard units to match and measure length
e. Recognising the need for uniform units in order to make fair comparisons between lengths
f. Selecting from a variety of counting strategies in order to calculate the length more efficiently
g. Choosing and using appropriate non-standard/ standard units for measuring length

h. Choosing and using a referent or go between when direct comparison cannot be made
i. Developing a sense of accuracy when using non standard measuring tools
j. Identifying the need for base line when comparing lengths
k. Repeating a unit without gaps and overlaps, placing units end to end
l. Developing a sense of estimation
m. Recognising the need for standard units to compare and communicate across settings
n. Recognising and exploring the pattern in metric units and their relationship to the Base 10 system
o. Converting between the different units when the task requires

(From Tania's case record)

Designing investigative tasks

In using the Growth Points to design investigative tasks, Tania ensured that each task had different entry points to allow students to employ a range of strategies, which of course depended on their prior knowledge. A range of materials was made available to cater for these different entry points. Figure 17 illustrates how Tania used the Growth Points as a conceptual framework when designing tasks.

The short-term focuses were linked to one or more starting points and demonstrates that Tania knew exactly what she wanted the students to know after they had completed their investigations. Tasks that were set at the beginning were just starting points, set up to begin the activity. Tania asked individual students further questions during the investigations to extend the task to more complex levels. For example, one of the tasks required the students to measure strips and to look for a pattern between centimetre and millimetre measures. As the students worked through the task, Tania moved them onto working out the relationships in the metric linear measurement system.

In the investigative tasks students collected data, searched for patterns, made conjectures, and analysed and verified their findings in order to draw conclusions based on the data. Some of the tasks were contextualised tasks, set within a 'real life' context. By providing students with contextualised tasks, Tania supported her students in applying the mathematical knowledge they had investigated. She also involved the parents in her attempt to encourage the students to appreciate the relevance of mathematics.

Figure 17. Tania's use of Growth Points in her planning

Example 1

Short-term focus	Starting points/ experience/task
Developing an awareness of linear measurement as an attribute. Classifying, comparing, matching and ordering linear measurement through direct comparisons. Distinguishing between different aspects of linear measurement such as width, length and height perimeter. Choosing and using standard units to match and measure length.	The children were given a variety of irregular and regular polygons. They chose 3 regular or irregular polygons, selected an attribute, ordered them and verified their order. *Is there another way to order your shape?* (Tania's case record)

Example 2

Short-term focus	Starting points/ experience/task
Developing a sense of estimation. Recognising and exploring the pattern in the metric and the relationship between these units and the Base 10 system and converting between the different units when the task requires. Choosing and using standard units for measuring length appropriate to the task at hand. Developing a sense of accuracy when using non-standard and standard measuring tools.	There were 2 sets of 20 strips. One set was cut to into lengths of 10, e.g., 10 cm, 20 cm, 40 cm, 90 cm, 110 cm. The other set was cut to the nearest centimetre, e.g., 1 cm, 5 cm, 7 cm, 33 cm, 74 cm. The children measured 20 of the strips in millimetres and centimetres. They explored the pattern in the metric units and the relationship between these units and the Base10 system. (Tania's case record)

Supporting continual assessment

A major theme emerging from both the classroom observations and Tania's case record data is Tania's continuous assessment of her students, based on a combination of work sample analysis, observations of students, and conferencing. Included in her planning framework was a set of questions that she used during conferences to determine students' understanding of the concept.

During a conference Tania used questions to assess individual students' thinking. Their responses then informed her next question or the next investigation/task. The assessment information informed new directions for individual students, as well as for whole-class programming. This is evident in the students' work samples and descriptions of the different pathways they took within the tasks set (see 'Using the Growth Points to Support the Designing of Investigative Tasks', described previously). There is meaningful evidence of the use of Growth Points as a conceptual framework to inform assessments and to design subsequent questions and investigations to challenge and support the direction of students' learning. Excerpt 13 illustrates how Tania used the Growth Points to assess and link Sam's prior knowledge to inform particular types of questions that would facilitate the building of new knowledge.

Excerpt 13

Tania	*What is area?*
Sam	*The space inside the shape.*
Tania	*So I see you are counting the whole squares inside the triangle?*
Sam	*Yes.*
Tania	*Why not the parts?*
Sam	*Are they necessary?*
Tania	*Well, are they part of the area?*
Sam	*Yes.*
Tania	*So do they need to be counted as a measure of the area?*
Sam	*Yes, but I'll have to cut them out and slice them up to make whole squares?*
Tania	*That sounds like a good idea.*

Sam made 3 square centimetres from the parts and added this to his recordings.

The Growth Points related to this excerpt highlights the importance of students *developing a sense of accuracy dealing with parts of units.* Tania observed Sam's omission of parts of units then questioned him to focus and challenge his thinking about what the attribute of area is and how to more accurately measure by including the parts of units. Tania encouraged him to decide for himself how he might deal with the part units she then followed his thinking and affirmed his decision. Tania used scaffolding techniques to

help Sam realise that areas do not always come in whole units but can be place together as parts in the count.

Excerpt 14 is a typical example of how Tania used the Growth Points in her questions to support students in developing a deeper understanding of mathematical concepts. In this excerpt Lynne was investigating the relationship between centimetres and millimetres using a range of strips cut to different lengths. Tania noticed that Lynne had collected the data (see Figure 18) but had failed to use it when searching for a pattern or relationship between the units. Tania wanted Lynne to recognise that she must multiply by 10 to convert centimetres to millimetres rather than saying 'add a zero', but Lynne was perhaps confused by Tania's questioning.

Excerpt 14

Tania	*Can you find a relationship between centimetres and millimetres in your data?*
Lynne	*You just add a zero when converting centimetres to millimetres and when converting from millimetres to centimetres you take a zero away.*

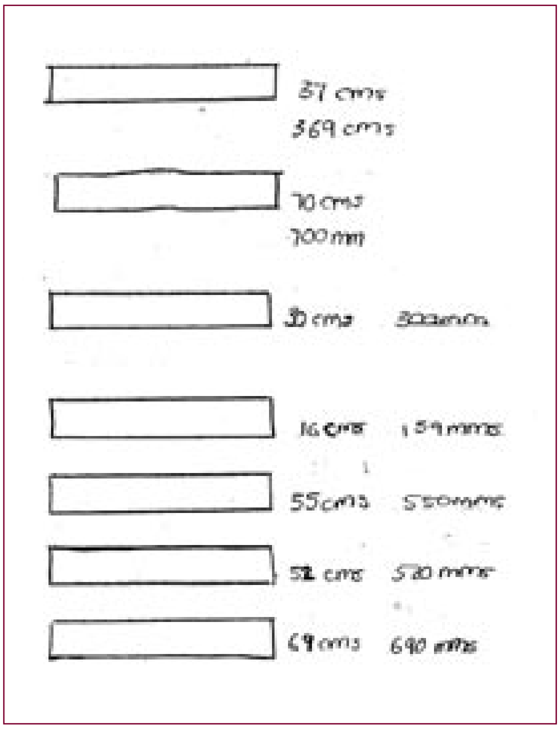
Tania points to 50 cm.

Tania	*What is this in millimetres?*
Lynne	*500 mm.*
Tania	*So what would this be in millimetres? [pointing to 40 cm.]*
Lynne	*400 mm.*
Tania	*So would 55 cm be 550 mm?*
Lynne	*Yes.*
Tania	*Think to your self, how many millimetres in a centimetre, then? Can you see a relationship between your numbers?*
Lynne	*Yes, it adds a zero.*
Tania	*Check with a calculator. Does adding a zero work.*
Lynne	*No.*
Tania	*Why do you think? What process could you use for it to change from 170 to 1700?*
Lynne	*I'm unsure.*

(Tania's case record)

Figure 18 shows Lynne's data from investigating the relationship between centimetres and millimetres.

Figure 18. Lynne's data from investigating the relationship between centimetres and millimetres



Although Lynne had observed a pattern, Tania felt that she had not yet linked this pattern to the relationship between centimetres and millimetres. From this assessment Tania responded by immediately designing a new task (see Excerpt 15) to encourage Lynne to reflect on this relationship in order to link the pattern in her data to it.

Excerpt 15

Tania	*Draw 5 lines with different lengths in centimetres. Then see if you can see how many lots of 10 mm there are in each. Then go back to my question … what process can you use to convert millimetres to centimetres and centimetres to millimetres?*

Tania left Lynne, who then drew a 10-cm line and carefully marked in the millimetres in lots of10. She then wrote:

1 cm = 1 lot of10 millimetres

2 cm = 2 lots of 10 millimetres …

9 cm = 9 lots of 10 millimetres.

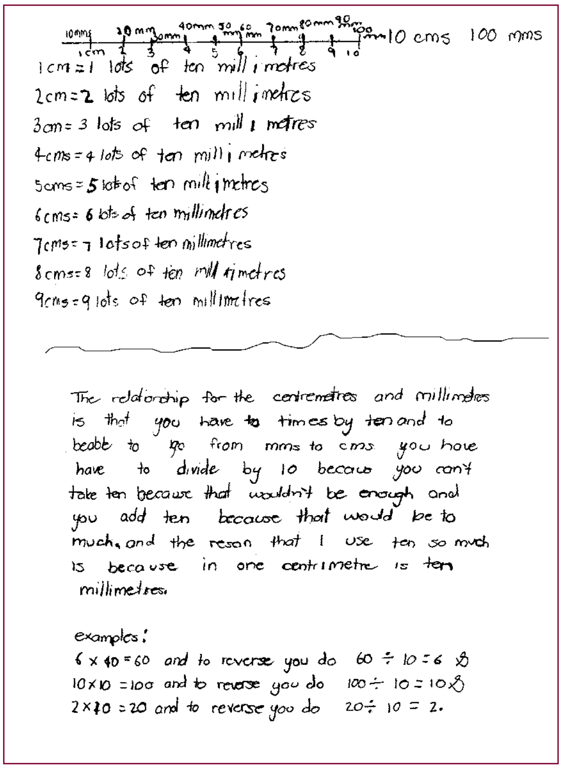
Lynne explained to the girl next to her that she thought that *for every 1 cm the lines go up by 10mm.* Tania overheard Lynne and returned:

(Tania's case record)

Tania	*What do you mean by that, Lynne?*
Lynne	*I think the relationship is that for every centimetre you times it by 10 because there are 1 mm for each centimetre. So if you have 6 cm then there will be 6 lots of 10 mm so you times it.*
Tania	*Well, what about if you had 60 mm … how would you work out how many centimetres that would be?*
Lynne	*You can't take 10 because that would not be enough … you can't add 10 because that would be too much and the reason is there are 10 mm in a centimetre so you times to change centimetres into millimetres so would that mean that you divide by 10?*
Tania	*That's right. Can you write down your thoughts on that with some examples?*

Figure 19 shows Lynne's understanding of the relationship between centimetres and millimetres.

Figure 19. Lynne: Relationship between centimetres and millimetres



Tania identified Lynne's understanding of the relationship between the metric units as a Growth Point. Having this conceptual framework in mind, Tania had provided the scaffolding for Lynne to think through the relationships and construct her understanding. Tania's teaching was therefore underpinned by the Growth Points — she used them as a conceptual framework to help her to plan and develop the mathematical ideas she wanted the students to investigate, the tasks she designed, and the sorts of questions she used to ensure that the mathematical knowledge the students were constructing as they investigated was challenged. Through discussions with the students Tania ensured that they shared common understandings.

Supporting LBOTE students

Tania had 10 LBOTE students in her class. As stated at the beginning of this case analysis, the school provided a lot of support for LBOTE students. When grouping students Tania made sure that there was at least 1 person in the group who could model the language and who read well. Tania also encouraged students who spoke a language other than English to use their own language, for example, Vietnamese, to explain an idea to another student who spoke the same language. Another requirement in Tania's classroom was that students had to express their thinking in their own words. They all had to express themselves in English but they did not have to be grammatically correct. The students' ability to express themselves in formal mathematical language improved as the year progressed. From the analysis of the students' workbooks it was evident that initially students consistently documented their thinking and findings using their own informal language. Generally, however, the students moved from informal to conventional language as the unit developed.

Figures 20 to 22 are typical examples of LBOTE students' documentation using their own informal language to describe the patterns in their data and make generalised statements based on the patterns they have observed. In these examples it can be seen that all 3 students – Harrah, Trang and Nhi – did the same task, but they recorded their findings in their own way, using their own language.

Figure 20. Documentation of patterns in area data: Harrah

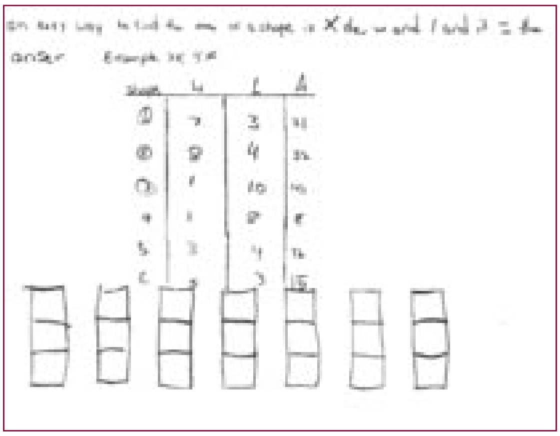


Figure 21. Documentation of patterns in area data: Trang

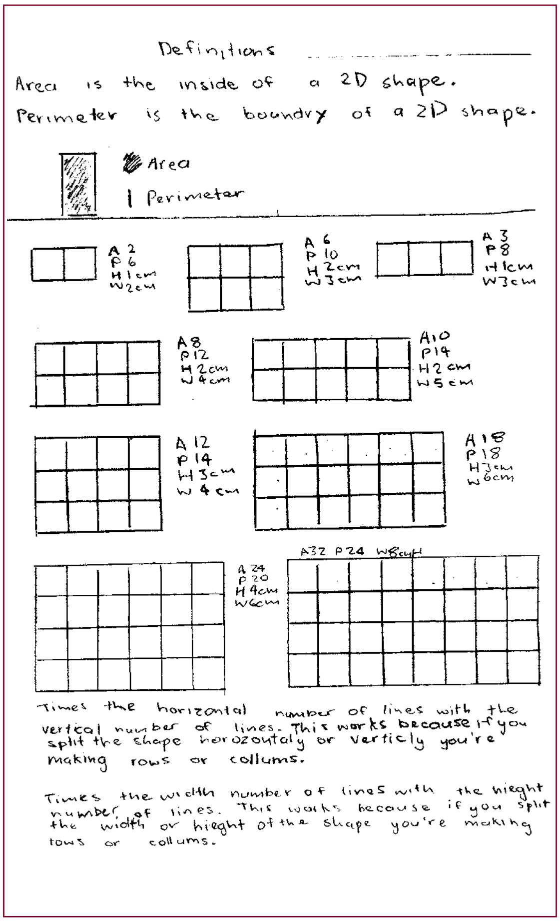
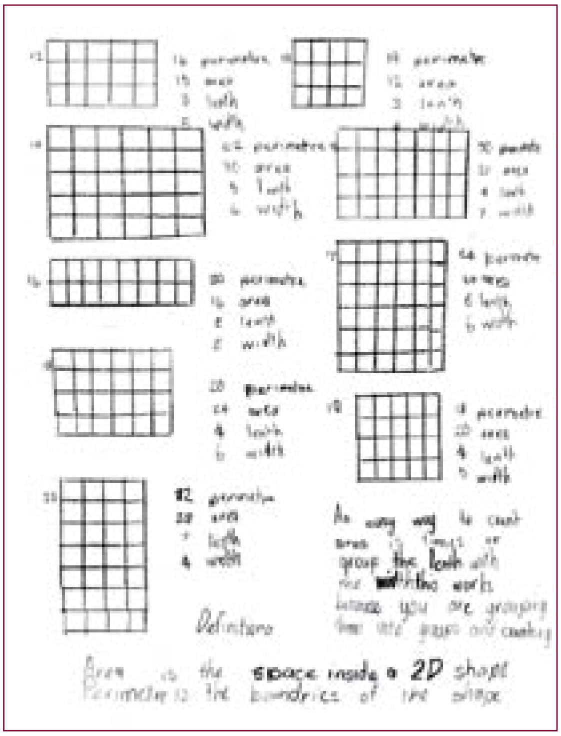


Figure 22. Documentation of patterns in area data: Nhi



Summary

Tania's classroom reflected her beliefs of how students learn mathematics/numeracy. The social and physical settings of the room were organised to support collaborative and co-independent working style enabling students to share and construct mathematical ideas. Tania established herself as a facilitator rather than the only person with the knowledge, thereby creating an atmosphere where students respected each other's ideas and were willing to collaborate.

The physical setting of the room and the class protocols also supported the social setting as the students constructed understandings in mathematics/numeracy within a social constructivist environment. Students worked in mixed ability and mixed gender groups. As there were a large number of LBOTE students in the class, Tania ensured that at least 1 student in the group was able to read English and the classroom displays included useful mathematical vocabulary.

Analysis of the pattern of interactions in Tania's class revealed that she kept whole-class interactions to a minimum, using most of the session time to conference students either in one-to-one or small group settings. When Tania interacted with small groups she questioned all the members of the group, and all the students were given the chance to join in the discussion. This demonstrates Tania's preference for having direct contact with individual students.

Tania used conferences to facilitate learning during the mathematical experience part of the session. Before starting a conference she established each student's current understanding and knowledge so that she could support them in bridging the gap between what they already knew and what she intended them to learn. She used questions to establish the students' thinking and the strategies they were using. Her questions required students to verbalise their thinking, and in doing so, to reflect on the choice of strategies they were using when constructing deeper mathematical understandings. The students' responses determined Tania's next question, which was often designed to challenge the students to move to greater and more sophisticated level of understanding of the mathematical ideas, or to rethink strategies, leading them to find more effective ones.

Through conferences Tania assessed individual students' knowledge and provided them with feedback through conversation. She also used the work samples as another means of assessment, looking at the students' written thoughts. As Tania focused on individuals she was aware of how each student was making sense of the mathematical experiences and what their needs were. Because the students communicated their understanding using their own language as well as the conventional mathematical language, Tania gained access to individual students' thinking and understandings. Either in a written form or orally, students had to explain their thinking and this forced them to think through their thoughts in a logical sequence.

Tania used the Growth Points as a conceptual framework when planning a unit of work and when assessing individual students' growth. Growth Points also assisted her in the planning of investigative tasks. The investigations Tania gave her class were open enough for all students to be able to commence, allowing students to start either at a very basic level or at a higher level, depending on their prior knowledge. Growth Points were used to help Tania to think through the conceptual ideas students needed to understand as they explored and built mathematical knowledge.

Case Study 2: Zoë

Zoë had been teaching for 18 years, the last 4 of which were at St Stephanie's school. In the year prior to the project, Zoë was involved in numeracy/mathematics professional development provided by Catholic Education South Australia. Working from a constructivist philosophy, the professional development used an action research model where teachers and facilitators undertook classroom research of student learning in mathematics, and its possible impact on teaching strategies. At the start of the current research, Zoë had had a year's experience in working in this style. She had not, however, used measurement Growth Points before the research.

School setting

St Stephanie's School is located in a suburb identified as low socio-economic in the Adelaide Social Atlas (2001). The school has a large intake of LBOTE students from a variety of ethnic backgrounds and has a strong focus on LBOTE perspectives. In the year of the research the school population of 390 students occupied 13 classes ranging from Reception to Year 7. The school had 25 staff of whom 13 were teaching staff. There were 9 composite classes, although Zoë's class of 29 students was a straight Year 3 class. Two students had special needs and 4 were designated LBOTE. The LBOTE students in Zoë's class received 40 minutes support per week from a specialist LBOTE teacher. A considerable number of the students were on school card.

Physical setting of Zoë's classroom

Zoë's Year 3 class was in a transportable/demountable building alongside other Year 3 classes which were housed in the same type of building. The physical space in Zoë's classroom was set out for students to work together in groups, but there was also provision for the teacher to work with the whole class. The layout, resources and displays in the classroom seemed significant in creating a supporting environment in assisting students as they made sense of mathematics/numeracy.

Layout

The physical layout of Zoë's classroom evolved as the year progressed. Initially, Zoë felt that the students worked better in pairs than in small groups for investigations so the classroom was set for whole-class discussions and for paired work, with the desks arranged in staggered rows around the room, all facing the front. At the front of the classroom, next to the whiteboard, was a large cleared seating area. This was where the students came together as a whole class for introductions or sharing sessions. The students who wanted more space for their investigations or preferred working on the floor also used this area. Zoë also used this space when she worked with small groups of students.

As the first term progressed Zoë found the physical layout of the classroom hindered her facilitation of individual student learning. When she tried to move around the room to work with individual students at their desks, she experienced difficulty reaching the students and when she did she could only stand behind them. She felt this made it difficult to



‘connect’ with these students and to affirm or challenge their work. Zoë also realised that Robert, who had a specific learning difficulty, often placed himself in the most difficult place to access.

After sharing and discussing several observations with the project researcher, she decided to re-arrange her classroom to encourage more peer collaboration. This new arrangement created space for her to access all students and to get down to the students’ level to make eye contact, thereby encouraging a freer flow of discussion between her and the students. Having to work in small groups also provided the students with more than one other person with whom to share ideas and strategies.

Resources

Zoë placed a variety of resources either on the students’ desks or on the floor in front of the classroom. Students made choices from a range of materials that always included both standard units (for example, centimetre square blocks) and non-standard units. The tasks were written on the board at the front of the room. This board also displayed the list of mathematical terms that had been drawn up during an introductory session or added to during a *sharing time* session.

Displays

There was always a display of language/terminology, ideas or diagrams on the board at the front resulting from brainstorming or sharing sessions. As new language, ideas and strategies were discovered or discussed they were added to the lists. Zoë always referred to these displays during the introductions, the sharing sessions and the conferencing, highlighting their use as a resource for supporting and building mathematical language/terminology and ideas. During the research, students were observed referring to or showing others the display when unsure of a correct spelling or terminology.

The display on the board at the back of the room comprised students’ work samples from their investigations in area/linear measurement. These were accompanied by descriptions of strategies used by the students when exploring the task. As the unit on area progressed the students revisited their original descriptions of area and rewrote them to better suit their current understandings. The purpose of the display was two-fold – Zoë used it as a means for valuing individual students thinking as well as providing visual models of different strategies, ideas and language. As well as students’ work, the display included lists of useful words, and class and individual definitions of area. The displays in Zoë’s classroom also included manufactured displays for supporting the students with their tables’ knowledge.

Social setting of Zoë’s classroom

Grouping strategies

Zoë regularly rotated her group memberships, making sure that at least two new students joined a group each time. The groups were mixed ability and varied in size from 4 to 8 students. Although Zoë selected the groups, the students were free to choose their seating arrangements. There was an empty space at the front of the room where some groups chose to work. Zoë noted the change of grouping as part of her facilitation in her case record:

> I have changed the grouping of the students so they are of mixed ability and are working alongside at least two new peers.

She felt that students needed to learn to work with all members of the class. Changing groups also provided an influx of new ways of thinking as students worked together on investigations.

Protocols – encouraged behaviours in the classroom

The protocols operating in Zoë's classroom were centred on encouraging students to work together, to discuss and share strategies and to support each other with mathematical ideas and language. The students were encouraged to think through the tasks, then to collect data and to use it to work out their solution. Students had to keep records of their ongoing thinking and had to record their own solutions and also their reasons behind the solution.

Following are two examples from different observation sessions that illustrate Zoë's emphasis on encouraging this behaviour.

> Today I want you to look through your data that you collected … Remember we feel it is important to explain your answer. Look at your data, and think about how you got your data and how you got your answer.
>
> I want to see your thinking from the beginning of the activity right through to the end to see if there are any changes in what you are thinking. Use diagrams or tables if you need.

The classroom rules were designed to support this process. In one of the observations Zoë reminded the students:

> *If you have trouble with the task today, remember, ask someone else in your group, put your name on the board and go back to your seat and get on with something else.*

Zoë also ensured that every student had an opportunity to share their ideas and strategies with the group. In an attempt to ensure equity, Zoë kept a checklist where she recorded when students had presented their work to the rest of the class. Students were encouraged to document their thinking using their own language and in every observation there were numerous examples of Zoë exploring links between the students' language and conventional mathematical language.

Classroom interactions

As indicated in the methodology, data was collected fortnightly and as a result eight 8 observations were made in Zoë's classroom. At the beginning of each mathematics lesson, and again at the end of each lesson when she regrouped the students, Zoë interacted with the whole class. During the lesson Zoë conferenced students in their groups on a one-to-one basis. The physical organisation of the classroom and the social setting, especially the protocols Zoë put into place, supported this level of interaction.

Students' working styles

Students were organised in mixed ability groups. The protocols were set up to enable students to discuss and share their ideas and strategies as they investigated the mathematical concepts and solved problems.

In a mathematics lesson, students were either engaged with the same task or different groups of students worked on different tasks. For example, one group could be exploring the relationship between the 4 sides of rectangles while another group could be measuring strips to find the relationship between centimetres and millimetres. However, all students had attempted all of the activities by the end of the unit of work on linear measurement.



Interestingly, most of the time students did not go about their investigations in the same way because the strategies each used depended on their prior knowledge.

In all of the 8 sessions observed, the students worked either collaboratively or co-independently, although the co-independent style of working was used more frequently. These two styles of working resulted from the settings, the task and the facilitation strategies that Zoë employed in her mathematics lessons. Zoë acknowledged that there were times when she wanted the students to work as a group and other times when she wanted them to work individually. She did not, however, prescribe their styles of working. Excerpt 16 and Figure 23 illustrate how these modes of working support students' thinking.

Collaborative style

Excerpt 16 is an example of students working collaboratively to measure an area using a one square metre shape. In the previous lesson the students had constructed a square metre, which they could use to measure games in the playground space.

Excerpt 16

In the cubby house 4 students laid one square metre on the floor. They needed more 'paper' to cover the rest of the floor space. Following is part of their discussion as they tried to work out its area.

James *OK, we've got one square metre and add this [the length from the wall to the square metre].*

Robert was talking to himself in an attempt to solve the problem while James was moving the square metre around to get it to fit onto the floor.

Robert *But this is a square metre.*

Sally *I've got an idea what we can do, we can get some more newspaper and put it around here and then we can add up how much newspaper that is.*

Robert *OK, you have to get more newspaper.*

Sally *[to another student] We'll get more newspaper.*

James *This is just one metre [referring to the square metre].*

Robert *[talking to himself] OK, that's just one metre … and it's got all those [putting his hands in the air and showing that he is thinking of the 3D nature of the cubby].*

James *Yes, it's how the cubby is all shaped, you know how it is … it's got all those dents and crannies.*

James *I'll say it's a metre.*

Robert *It's a metre, OK [reading off the tape measure].*

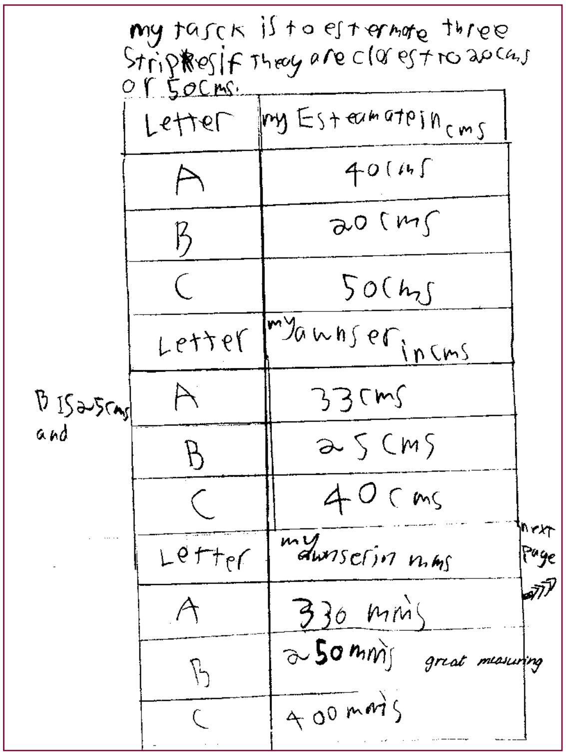
James *Our square metre here is not a square metre.*

The students worked collaboratively to solve the problem. They shared ideas and worked but they all recorded their measurements, strategies and findings in their own way.

Co-independent style

Figure 23 shows examples of the recordings of 2 students – James and Marc – who worked co-independently. Although they discussed ideas and the strategies they used, the recordings of their findings were different. The 2 students recorded their findings in their own ways and Zoë responded to them individually. As Zoë conferenced the students she assessed their ' thinking individually.

Figure 23. James (left) and Marc (right): Independent recording of investigation



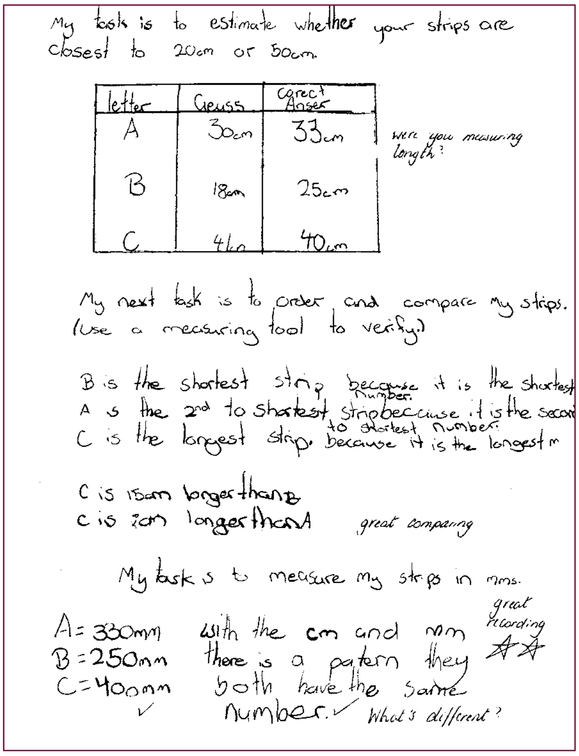
my tasck is to estermate three strips if they are clorest to 20 cms or 50cms.

Letter	my Esteamate in cms
A	40 cms
B	20 cms
C	50cms
Letter	my awnser in cms
A	33cms
B	25 cms
C	40 cms
Letter	my awnser in mms
A	330 mms
B	250 mms
C	400 mms

B is 25cms and

next page

great measuring



My task is to estimate whether your strips are closest to 20cm or 50cm.

letter	Geuss	Corect Anser
A	30cm	33cm
B	18cm	25cm
C	46cm	40cm

were you measuring length?

My next task is to order and compare my strips. (use a measuring tool to verify)

B is the shortest strip because it is the shortest number.
A is the 2nd to shortest strip because it is the second to shortest number.
C is the longest strip because it is the longest n

C is 15cm longer than B
C is 7cm longer than A

great comparing

My task is to measure my strips in mms.

A=330mm
B=250mm
C=400mm

with the cm and mm there is a patern they both have the same number.

What's different?

great recording

The mathematical experience

The mathematical experience Zoë organised for her class comprised 3 inter-linked yet distinctive moments within the full session – the introductory *setting the scene and tone, Students' investigations* and *sharing time.* The following section discusses how Zoë supported students' learning within the framework of these 3 periods.

Setting the scene and tone and sharing time

Each of the mathematics sessions observed started with the whole class gathered at the front of the room near the white board. The purpose of the introductory part of the session *(setting the scene and tone)* was to involve students in discussions and brainstorms about their own and conventional mathematics ideas and strategies that they might find useful when investigating or solving a problem. Usually, when Zoë introduced the students to new concepts, the discussions focused on the students' use of ideas and language. For example, when discussing *area,* Zoë supported them in linking their knowledge to the mathematical concept of area and its relationship with perimeter. Throughout the presentation Zoë continually summarised the ideas and strategies she deemed useful for the students to think about.

The language discussions encouraged the students to explore the links between their own language and the conventional language of mathematics. They also brainstormed the words they used and those associated

with conventional mathematical terminology. These words were added to a list that was kept on display throughout the unit of work. The list was often added to and referred to by Zoë and the students throughout the investigations and *sharing time.*

In both the *setting the scene and tone* and the *sharing time* Zoë encouraged the students to reflect on their selection of mathematical processes and methods. Students were invited to the front of the class to share the strategies they used in the investigation and to explain the thinking behind their recordings. During these periods Zoë purposefully selected different students to highlight different strategies she felt could be supportive of all students' learning.

Zoë kept a record to ensure that all students were provided with opportunities to share their own ideas with the group. This process of sharing and verbalising ideas was not only supportive of students but enabled Zoë to convey to the students that she valued their ideas. The verbalisation of their ideas to others may have helped the students to internalise and reflect on their own thinking. It also provided models of thinking, questioning and strategies for other students. Listening to other students' reports could have been a means of affirming or challenging the students' thinking as well as encouraging the students to acknowledge that there was more than one strategy for solving a problem.

Zoë facilitated these conversations by asking questions of individual students or the group. These questions seem to draw out the important points Zoë considered all students needed to know and understand about the piece of work. Sometimes within these whole-classroom interactions Zoë picked up on mathematical ideas and challenged individual students or the whole class. In these whole-class interactions Zoë also encouraged and discussed the use of mathematical processes such as collecting data, organising data collection through the use of tables and drawing conclusions based on the collected data.

During the *setting the scene and tone* introduction Zoë always reminded the students of the range of materials at their disposal, encouraged positive behaviours in the classroom and stressed the importance of a positive working environment where students could work collaboratively or co-independently. The *setting the scene and tone* also provided opportunities for Zoë to create links between previous sessions and related mathematical ideas. Predominantly teacher-led, these periods as well as *sharing time* were discussions or conversations (public conferences) between Zoë and the whole class or a student, while the other students in the class listened.

Excerpt 17 shows how Zoë supported the students in the linking of their own understanding of area to the conventional concept.

Excerpt 17

Introducing area

Zoë	*Today we are going to talk about area. Have you heard this word before? Where have you heard it?*

The students started to name some areas around the home and schools that they know.

Lucy	*The laundry area.*
Zoë	*Do we always talk of the laundry area?*
Student	*Yes.*
Hannah	*The bathroom, and toilet area.*
Student	*The play area.*
Robert	*A room or an object.*
Zoë	*Any other word.*
Student	*Places.*

The students continued to name areas around the home or school.

Zoë *How do you know it's an area? What makes them an area? … What do you think about when you think of area, the laundry area, the play area or the school area?*

Student *There is some thing surrounding it like a door or a fence.*

Zoë *OK, surrounded, I like this word surrounded, surrounded by fence or marking or … someone had a really good word here.*

Elizabeth *Something surrounding, around it like a fence, making a boundary or a wall.*

Zoë wrote this information on the board behind her.

Zoë *OK, surrounded, I like that, surrounded by a fence or marking.*

Zoë *Someone has a good word over there.*

Student *Line, boundary.*

Student *Boundary.*

Zoë *They are all boundaries, can I include all of these (fences and all) under boundary.*

There was a 'yes' and a few students said 'no'.

Zoë *When we are talking about surrounded by a boundary what are we talking about? It seems that there is a word. What are we talking about? What is surrounded by the boundary?*

Student *Might be a room or an object.*

Zoë *Is there a word that will cover a room or an object?*

Student *Places.*

Zoë *Places surrounded by marking or boundary, walls, any other word we can use here? Places is fine. OK, I am going to put places for now, a place surrounded by marking, fence wall etc. When we come back from our walk we will go back to our definition of an area and see if there is anything we would like to change, and what things we would like to keep. There are some really good words in there. Now you are going to have a lot more ideas as we go around and find out the different areas around the school.*

Zoë used the students' descriptions of area to support them in linking their informal/prior knowledge to the conventional description of *area*. As the students described their understanding they also linked it to *perimeter,* learnt in the previous unit of work. Excerpt 18 illustrates a student sharing his strategies, mathematical understanding and language with the rest of the class.

Excerpt 18

Nick is invited to the front to share his investigations. He has been using a transparent centimetre square grid to work out the area of his shapes in order to compare which target shape would cover more space on the wall.

Zoë *Can you show how you're measuring your area and what you are doing with the parts of squares?*

Nick *I'm dotting the parts that can make a whole square.*

Zoë *So how much space is that?*

Nick *Half.*

Zoë *How much of a square have you got up there? [Zoë points to other parts of his recording]*

Nick *Half.*

Zoë *So you're marking them to say that those two parts make a whole?*

Nick *One whole.*

Zoë: *When you have parts of squares you can put parts together that are closest to a whole square.*

Nick nodded.

Zoë *You need to find some way of keeping track of these so you might dot or number them. Will you be able to say the number of squares is exact?*

Students in unison *No.*



Zoë	*Or you could say the number is between say 27 and 29 squares. You can't be one hundred percent accurate when counting parts of squares but you can be close to.*
Student	*How do you know those two parts will make a whole?*
Nick	*[looking at the parts] They look closest to halves.*
Zoë	*That's a 'best guess', that's why we talk about 'closest to'.*

(Zoë's case record)

During this public conference, Nick talked through the strategies he employed and the words he used to describe his findings. Some students in the classroom were able to identify with the strategies and thinking when working with parts of wholes and the words he used to describe their findings, hence their input.

Student investigation time and conferencing

During the *student investigation time* Zoë conferenced students individually. She also moved around the room answering students' questions, and ensuring that all students were on task and able to start the investigation. When Zoë was unsure of the students' thinking or the strategies they were using, she questioned them and her questions helped them to clarify their thinking in order to solve the problem.

Zoë averaged 4 conferences of 6–10 minutes each, per lesson. The time available for conferencing the students depended on the length of the introduction of the lesson as some *setting the scene and tone* periods tended to be longer than others. Zoë selected and kept a record of the students she conferenced in all sessions. Usually before starting a conference she observed the student at work in their group to establish the strategies being employed as well as the student's understanding of the mathematical concept being explored.

Throughout the conference, as Zoë assessed the student's thinking, she affirmed the responses and encouraged the refining of strategies. Zoë moved on once she was sure that the student's thinking was being challenged. Excerpt 19 is an example of a conference between Zoë and a student.

Excerpt 19

Zoë was talking to four girls who were working out how much playground space the Four Squares game covers. On the calculator the girls put in 3900 x 3900 and got a very large number: 15210000. They all laughed.

Zoë	*So what did you work out?'*
Student	*Well it equals that.*
Zoë	*What is that number?'*
Student	*3900 times 3900.*
Zoë	*So tell me how you worked it out?*
Patricia	*We measured this side down and we got 3900 and we measured that side and got the same number as that side.*
Zoë	*Were you surprised to get the same answer?*
Patricia	*No. I wasn't surprised … I knew.*
Zoë	*Then what did you do?*
Patricia	*Then we multiplied them.*
Zoë	*So why did you multiply, Patricia?'*
Patricia	*Because I always do and I don't know why.*
Zoë	*So what did you get when you measured the sides?*
Patricia	*3900.*
Zoë	*3900 what?*
Patricia	*I think millimetres.*
Zoë	*OK, how much would that be in metres?*
Patricia	*I don't know.*

Zoë *Is there a way you could work that out? How many millimetres are there in a metre … do you know?'*

Patricia *A thousand.*

Zoë *So how many metres do you think you've got there?*

Patricia *Three.*

Zoë *Exactly 3?*

Patricia *No.*

Zoë *So, can you have a look and see if you can work it out?*

Patricia went back to the tape measure that was on the ground and showed the teacher the measurement she had.

Patricia *Almost 4.*

Zoë *So it's just under 4 metres long is it? So what do you think approximately the area is in square metres?*

This excerpt exemplifies how the teacher supported Patricia in working out the area of the Four Square game. Using a series of questions Zoë encouraged Patricia to reflect on her strategies, specifically her choice of unit in order to continue with her investigation. Zoë's questions required Patricia to call on her prior knowledge of the relationship between linear measures – millimetres and metres – in order to make the numbers, and therefore the task, manageable.

Public conference

During the *setting the scene and tone* and all *sharing time* periods Zoë engaged the students in public conferences. As described in the section on interactions, students were asked a series of questions that required them to explain their strategies and to verbalise their thinking to the rest of the class. Zoë summarised and highlighted important points and ideas she wanted to stress, and to encourage their use.

Excerpt 20 is an example of a public conference during a *sharing time.* As with any conference, the interaction was between the teacher and a student. In this instance, the student is sharing her thinking and the strategy she employed with the rest of the class.

Excerpt 20

Zoë *Let's have a look at this envelope here, what part of the envelope are you finding the area of?*

Penny *The middle and the front of it.*

Zoë *The middle and the front of the envelope?*

Penny *The front.*

Zoë *The front of the envelope … we are finding the measurements of the whole space and we know what the area looks like … you have called it envelope A. You have got some dots here and here. What are they for?*

Penny *First I counted across and then down.*

Zoë *Why did you do that?*

Penny *Then we can times.*

Zoë *How do you know that you can times those?*

Penny *Because if you count down.*

Zoë *What are you counting when you count down?*

Penny *Groups.*

Zoë *Groups!*

Zoë now addressed Jake who was in the audience.

Zoë Can you see where the groups are, Jake?

Zoë summarised what Penny had said as she was trying to ensure that the rest of the class was attentive as she modelled a good strategy.

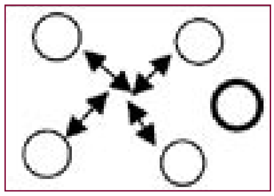
Zoë	*See! Penny has counted 12 across in a row and each row is a group because all the rows are the same size. Penny found a quick way of working out how many squares there are altogether. All she will have to do is count how many rows and how many … [inviting Penny to complete the sentence].*
Penny	*How many down?*
Zoë	*What are you counting when you are counting how many down? You are counting …*
Penny	*Columns.*
Zoë	*Rows, OK, so these dots down here tell her how many groups she has got or how many rows and because she knew they were groups she knew that the maths was … times. So Penny, when you go back, can you explain why it was that you used times there – when you were counting down? You need to explain in your recording. Some of you might need to do this too. What was it you were counting when you were counting down?*
Zoë	*What was it about the shape that said to you that you could count rows or that you could count down. If this had been a shape with bits out here and other bits sticking out there, would Penny have been able to count groups like that?*
Penny	*I could have drawn a square and timesed all of the squares and then counted all the left over bits.*
Zoë	*You certainly could have done. So what Penny is saying is, if she can find a space where the columns and rows are the same size you can still count in groups but then she would have to add on the extra bits. We can't ignore them, can we?*
Student	*Because then you wouldn't get an exact measurement.*

This excerpt illustrates how Zoë used this public conference to model to the rest of the class the efficient strategies Penny had used to calculate the area of an envelope. At various points in the interaction Zoë summarised the important messages she wanted the students to gain from this interaction.

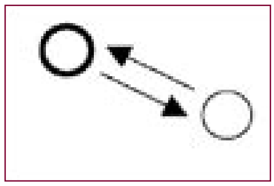
Interpretive summary of classroom interactions

Zoë's mathematics lessons have been divided into 3 significant, moments: introduction – *setting the scene and tone,* student investigation time and conferencing, and *sharing time.* Each of these had a different purpose in supporting students' construction of mathematical knowledge. The pattern of these interactions emerging from the data has resulted in a range of different categories. Following is a description of these categories of interactions, which were instigated by Zoë or by the students themselves. These categories are:

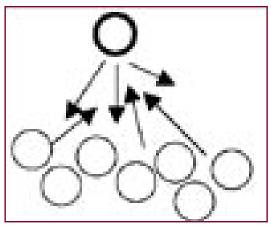
Category 1 Small group interaction with or without the teacher observing



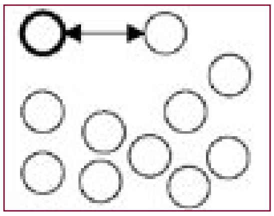
Category 2 One-to-one interaction (conference) between a student and the teacher



Category 5 Whole-class interaction with the teacher, occurring during setting the scene and tone or sharing times



Category 6 One-to-one public conference in a whole-class setting which, occurred during setting the scene and tone and sharing times



Each of these categories has been elaborated below using extracts from recorded conversations gained during classroom observations. What this study can say about classroom interactions is that it gave the teacher opportunities to find out about students' understandings, and prior knowledge of the concept in a non-threatening environment. As the classroom was set up for this sort of interactions to occur the teacher not only used it to support students as they construct their own understandings but also as a means for students to support each other in building shared understanding.

Category 1 Small group interaction, occurring within student investigations

The small group interactions occurred during the investigation time where students interacted with others – discussing strategies, making decisions and debating conflicting ideas. This type of interaction occurred during both collaborative and co-independent work.

Excerpt 21 illustrates a group of students interacting as they investigated which envelope would take more paper to make. When Megan requested assistance from her peers about ways to investigate and record her findings, Patricia and Linda supported her by sharing and discussing their strategies and ideas. During the discussion Linda challenged Patricia about her method of recording and as a result Patricia had to rethink her strategies. The design of the investigations and the encouragement of co-independent learning supported this type of interaction where students' learning could be challenged and supported by their peers.

Excerpt 21

Penny	*[asking Patricia] Can I go on the back of the envelope?*
Patricia	*Yes, you are allowed to open it up if you want to take the seal off.*
Megan	*I don't want to.*
Patricia	*You get the area by measuring across the envelope and then down the side because there are groups there.*
Megan	*Why didn't you stick your envelope on your recording?*
Patricia	*I didn't want to.*
Megan	*Then how did you show what envelopes you were measuring?*
Patricia	*I am using a table.*
Linda	*So what one is which?*
Patricia	*Here … large and small.*
Linda	*But these both look large to me.*
Patricia	*I can use ABC and label my envelopes the same way to match.*

Patricia then labelled each envelope and the corresponding envelope data on her table.

Student	How do you spell envelope?
Megan	If you can't spell it stretch it out en… ve… lope.
Linda	There it is on the board. Look up there.

Category 2 One-to-one interaction (conference) between a student and the teacher

Zoë always observed the student she wished to conference before she began a conference. Excerpt 22 shows how Zoë used one-to-one conferencing to advance learning.

Excerpt 22

Zoë was observing Stephanie who had taken several measures of the individual sides of the target shapes.

Zoë *How do these measures help you work out your answer?*

Stephanie *I can add up the sides to work it out but I don't know how to add up the fractions. One side is 14½.*

Zoë *[picking up a calculator] 0 .5= ½. So you can use this information when adding ½ on calculators. For 14½ you press 14.5. Remember when you have a ½ it is .5*

Zoë left Stephanie, but returned later to continue the conference.

Zoë *Stephanie, what have you found out so far?*

Stephanie *Two sides of the rectangle are the same as the other 2 sides.*

Zoë *What do you need to do about that?*

Stephanie *Times by 2.*

Zoë *Why don't you try that?*

Stephanie did what the teacher suggested and recorded 71 as her answer.

Zoë *What is that part of the rectangle that you are measuring that equals 71?*

Stephanie looked at Zoë without answering her question.

Zoë *So you put all the lengths into your calculator and added them together?*

Stephanie *Yes.*

Zoë *What is that measure? What part of the rectangle?*

Stephanie was still unsure.

Zoë *When we measure around the distance of the rectangle is there a word up on the board that could help you to name it?'*

Stephanie *Yes, it's the perimeter.*

Zoë *Could you use that word in your recording? You were going to say 71. You could say the perimeter equals 71cm. What data are you going to collect now? Have you been able to answer the question?*

Zoë assisted Stephanie in dealing with the fractional parts of her measures, which then enabled Stephanie to continue using her selected strategy. Zoë then further supported her in linking her concept of distance around the shape with the conventional, mathematical language. The one-to-one conference therefore created opportunities for Zoë to assess and respond to individual students' needs and growth.

Category 5 Whole-class interaction with the teacher, occurring during setting the scene and tone or sharing times

Zoë interacted with the whole class as a group, orchestrating and controlling the discussion herself. During such interactions different members of the group responded to Zoë's questions.

Excerpt 23 occurred during the *setting the scene and tone* period when Zoë introduced the concept of area. She involved the students in a discussion about the word area and where and how this term is used, which resulted in an initial description of 'area'. The class then went outside and used the description to identify areas outside the classroom.

Excerpt 23

Zoë concluded the lesson with a sharing time under the veranda with all the students seated around her. She questioned the students to bring out the meaning of area and how it could be measured. Students volunteered their ideas which Zoë summarised as the discussion continued.

There was a building site within the school grounds, referred to by the students as a construction area.

Zoë	*How did you know where the construction area was?*
Student	*Could see the building.*
Zoë	*Yes, is there something that clearly tells you where the area is that is different to other areas you have seen before?*
Megan	*It has a huge boundary.*
Zoë	*It has a huge fence making a boundary all the way around the building area, but which part is the building area? Is it the fence?*
Students	*No.*
Zoë	*Is the building area the fence itself?*
Students	*No.*
Zoë	*Describe what the area is, Stefan.*
Stefan	*It's in the middle of the boundary.*
Zoë	*Is it the whole space inside the boundary or is it just in the middle?' Who thinks that it might be the whole space inside the fence?'*

They all raised their hands.

Zoë	*Yes, the area is the whole space inside this fence, building on the construction area can happen anywhere inside that fence line.*

Within this type of interaction Zoë encouraged the sharing and discussion of strategies, mathematical ideas and language. The interaction provided the students with opportunities to explore and link their informal ideas and language to the conventions of mathematics. As a result, they created a common and shared understanding of area. The shared definition of area was rewritten on a large piece of paper, along with other meaningful words, and was displayed in the classroom.

Category 6 One-to-one public conference in a whole class setting, which occurred during setting the scene and tone or sharing times

In Excerpt 24, Zoë interacted with Stephanie at the front of the class while the other students observed. Sometimes she referred to the rest of the class for comments or answers to questions, but the conversation was predominantly between Stephanie and Zoë, with the rest of the group as an audience. The conversation focused on the students' work samples and the strategies used when investigating.

Excerpt 24

Zoë	*I saw some very interesting work and solutions today; some great ways of investigating your data. Would you like to explain what you found out today?*
Stephanie	*I got measuring tape and measured the shape and added up the sides on the calculator.*
Zoë	*Why did you get 71 cm?*
Stephanie	*I added all the sides.*
Zoë	*Yes, the perimeter.*
Zoë	*Did someone else come up with that word?*

Robert raised his hand and described it as the outside of the shape.

Zoë	*What do we mean when we use that word. What is the outside of the shape?*
Robert	*The border, finding the length of all the sides.*
Zoë	*Well done! Stephanie, I liked the way you worked that out, especially the way you worked out what to do with your left over parts of the units.*
Zoë	*Who else has a solution to the task?*
Zoë	*Stefan, would you like to share your work?*



This type of interaction was used by Zoë to model preferred strategies, to discuss and reflect on the students' use of language and to assist them in linking their own language to conventional mathematical language. During this type of interaction students shared strategies they had used with others in the class. The public conferences also served the purpose of valuing different individual students' thinking. Zoë kept a record of the students who had been invited to present to the class and tried to ensure over a period of time that all students had a turn.

Table 7 summarises the types of interactions in which Zoë engaged during the observed lessons. The pattern of interaction indicates that although Zoë valued whole-class interaction, the majority (70 per cent) of the interactions observed during any session were one-to-one.

Table 7 Summary of Zoë's interactions

Type of interaction	Setting the scene and tone	Students' investigation time	Sharing time
Category 2 One-to-one interactions between a student and the teacher	-	70%	
Category 5 Whole-class interaction with the teacher, occurring during setting the scene and tone and sharing times	15%	-	5%
Category 6 One-to-one public conference which occurred during setting the scene and tone and sharing times	6%	-	4%

In the one-to-one conferencing Zoë encouraged each student to think through strategies and to link their prior knowledge and language to the concept being investigated, thereby emphasising individual students' needs and growth. While Zoë conferenced individual students, the rest of the class worked either collaboratively or co-independently in their respective groups. The protocols operating in Zoë's classroom forced the students to support each other, consequently releasing Zoë to focus on individual students' growth and needs.

The whole-class interactions observed were either category 5 or 6. Both interactions involved the students in discussing, sharing and exploring ideas and terminologies associated with a new concept. Primarily, category 5 interactions were used for brainstorming strategies, identifying terminologies and developing a shared common understanding. Category 6, however, was used to model thinking, strategies and mathematical processes through a public conference.

Analysing the types of questions Zoë used during a conference

The sections on *Social setting* and *Mathematical experience* show that Zoë viewed social interaction as an essential part of developing understanding for all students. Using questions, Zoë orchestrated whole-class discussions and one-to-one interactions. As questions were a very important part of the interactions, we decided to analyse them to establish the sorts of questions Zoë was using and the sorts of responses she was receiving. Zoë's questions have been categorised according to the possible effect they may have had on students' learning. These categories are:

- reflecting and explaining current reasoning
- linking the students' own informal language and ideas with the conventional mathematical ideas and associated terminologies
- refining and building onto mathematical thinking and strategies.

Reflecting and explaining current reasoning

The information Zoë gained from these questions provided her with knowledge of the students' current understanding, which she then used to inform the direction and focus of her ongoing facilitation for individual students' learning. Excerpt 25 provides examples of this type of question.

Excerpt 25

Zoë	Why do you think you got that measure?
	(From observation 3)
Zoë	Patricia, what measures do you think you need to take?
Patricia	Area.
Zoë	OK, area … why do you think area?
Patricia	Because you are measuring the whole space, not just the edges.
	… … …
Zoë	So you are going to see how many rows and columns. What is it about the shape that tells you that you only have to do that?
	(From observation 6)
Zoë	So you had 6 rows of 12. What maths did you use to get 72?
Zoë	So tell me how you worked it out?
Zoë	So why did you multiply?
	(From observation 7)
Zoë	What are your reasons to divide them all? What will the answer be?
	(From observation 9)

Linking the students informal language and ideas with conventional mathematics

Zoë's questions encouraged the students to use their own language to describe their understandings. Through shared discussions she supported them in making links between



their own informal language and the conventional language and ideas of mathematics. Excerpt 26 illustrates Zoë's use of this type of question.

Excerpt 26

In a conference Zoë asked Stephanie to think through more effective ways to record her data. Zoë had observed that Stephanie had recorded her measures without naming the attribute measured. Following is part of this dialogue.

Zoë	*So you put all the lengths into your calculator and added them together?*
Stephanie	*Yes.*
Zoë	*What is that measure? What part of the rectangle?*

Stephanie was still unsure.

Zoë	*When we measure around the distance of the rectangle is there a word up on the board that could help you to name it?'*
Stephanie	*Yes, it's the perimeter.*
Zoë	*Could you use that word in your recording. You were going to say 71. You could say the perimeter equals 71cm?*

During a whole-class introductory session on area Zoë, together with the students, discussed their understanding of the term *area*, with the purpose of developing a shared understanding and a working description of area. The observations showed that Zoë worked with the students' language as she attempted to connect it to an initial class description of area. For example, 1 student had used the word *surrounded,* and another student described it as *a boundary*. In her subsequent question, Zoë picked up on these words as she attempted to link them to the conventional mathematical terminology. Excerpt 27 exemplifies this point.

Excerpt 27

Zoë	*When we are talking about surrounded by a boundary what are we talking about? It seems that there is a word.*

*(*From observation 5)

In Excerpt 28, which took place during a public conference in a *sharing time*, Zoë questioned the class about the different words they knew that might be useful when the measures were not exact. One student suggested '*best guess*'*.* Zoë picked up on the word and encouraged the students to think of other words to add to the list.

Excerpt 28

Zoë	*Who could give me some more language we could use when we talk about our best guess?*
Stephanie	*Between, and …*
Robert	*Approximately.*
Another student	*About.*
	(From observation 7)

Zoë's questions encouraged students to call upon the words they knew to describe their mathematical ideas. By incorporating the students' language in her questions she facilitated the links between the students' informal ideas and language and the conventional ideas and mathematical language.

Refining and building onto mathematical thinking and strategies

The questions in Excerpt 29 were derived from something a student had said or recorded. Zoë used these questions as a basis to encourage the student to think again about the conclusion they had drawn or the data they had collected. These types of question often resulted in the students reflecting on their data and using it to draw conclusions or to review their strategies.

Excerpt 29

Are you sure? Show me 10 mm.

If this is 10 mm can this possibly be 90 mm?

(From observation 3)

Before you thought it was a half a unit. If it is not ½ mm then what unit would it be half of?

(From observation 3)

What would happen if you tried to compare an area in triangles with an area measured in squares?

(From observation 6)

If you think the length of this edge is nearly 4 metres long, how can this one be 2 metres long?

(From observation 9)

In all of these questions it seems that Zoë was trying to encourage the students to think logically as a basis for the development of deductive reasoning. Working from the students' data, Zoë selected the information she determined as useful and correct to challenge the students' thinking. These questions followed the style: *If this is known and correct, can that possibly be correct?* Excerpt 30 shows typical examples of questions that appeared to encourage students to recognise the many different ways to solve a problem.

Excerpt 30

If you could solve it how do you think you would work it out?

(From observation 3)

Would that be the quickest way of doing it? What would be quicker?

(From observation 6)

So is there another way you could work it out?

(From observation 6)

These questions were posed either to individuals in a small group or to the whole class. They appeared to promote discussion about strategies and allowed students to make links between their preferred strategies and those of others, or to move to more sophisticated and conventional strategies. Questions in this instance seemed to remove the pressure of having to provide a correct answer and, in doing so, allowed the students to think and speak more freely. Zoë used this type of question when students were unsure how to answer or begin an investigation.

Although Zoë's questions have been categorised into different groups these questions are interrelated and together provide a supportive framework. This framework encouraged the students to work from their informal knowledge and language and then to link it to the conventional mathematics. The process involved the students in continually reflecting on their ideas and strategies which often resulted in them refining and building new understandings.

Using the Growth Points

Data were collected on how and when Zoë used the Growth Points during the course of the study. As with all the teachers participating in this research, Zoë used the pre-test data in conjunction with the set of Growth Points to analyse the range of thinking in her class while paying particular attention to individual students. She used the Growth Points to plan, design tasks, observe and assess students throughout this study. The Growth Points gave her a framework on which she could develop individual students' thinking as they engaged with the mathematics. Throughout the study we observed that Zoë's questions were based on the Growth Points; the reflections in her case study; the interview at the end of the study showed that she found them useful and supportive.

Planning units of work

Zoë used the Growth Points as a conceptual framework when planning units of work and the mathematical investigations. For every unit of work the teacher had to plan she outlined the key mathematical ideas that would be addressed. Zoë's case record shows that she listed the Growth Points (labelled as Mathematics) for mathematical ideas. The Growth Points outlined in the planning framework were used as guidance for her in understanding students' thinking in the area being investigated. Figure 24 shows a planning framework Zoë used when teaching linear measurement. The planning framework outlines the task, materials, language and possible questions.

Figure 24. Zoë's planning framework for teaching linear measurement

All students were given the same initial task. This task was designed to further observe ordering and comparing linear measures and the use of comparative language. It was also designed to move students towards the use of standard and non-standard units to order and compare.

Strand: Measurement
Topic: Linear Measurement

Mathematics

- Consider the purpose for measuring when choosing the most appropriate attribute and unit of comparison. Measuring to compare and order
- Generating, planning and using their own units of comparison to compare and match distances
- Recording and comparing measurements, and using standard metric units and standard measuring tools, to quantify distances
- Recording and reporting measurements in a variety of ways

Language

Order, verify, length, unit, millimetre, centimetre, metre

Materials

String, rulers, tape measures, carpenters tapes, shells, gum nuts, flip blocks, centicubes, longs, unifix cubes, frieze tape lengths – 10cm, 25cm, 50cm, 75cm, 100cm, 125cm, large polygons, curved shapes

Tasks

Order the strips
Verify the order
Estimate/ Measure using a different unit
Find different lengths on polygons/ curved shapes
Order the shapes

Questions

How much longer is that strip?
Could you measure this again and get the same answer?
Does it make a difference if your units are crooked?
What other units could you use?
How can you make your units easier to count?
What different lengths can you find?

Zoë listed possible questions she might ask individual students as she conferenced them. These questions alerted her to possible thinking she might see in her class, so that she could prepare appropriate responses and further questions in order to assist students during the investigation times, conferences, *sharing times* or when giving students feedback in their work books. For example, the task in this planning framework was to order several strips and to verify the order. Students were given different materials from which they could choose. Using rulers or tape measures students measured the strips, ordered and verified their order.

The teacher at this point may not be able to know whether the students are aware, for example, that other units could be used to measure lengths; that units need to be uniform; that there should be no gaps or overlaps; or that when using a tape measure the tape needs to be straight. The student could be using the ruler as a measuring scale without realising its significance, which often leads to misuse of the ruler and lack of understanding of what it means to measure.

The questions listed in Zoë's planning framework were used as reflecting tools to bridge the gap between the Growth Points and the task, aiming to bring out the unifying ideas in measurement. They also served as support for Zoë as she responded to individual students' needs. Zoë did not necessarily ask all of the questions and modified them according to the situation. Excerpt 31 illustrates how Zoë used the Growth Points to facilitate the strategies she wished students to use and the mathematical knowledge she wanted them to attain by doing the task. Sean was asked to reflect on the strategies that he was using to work out the area of the shape. Zoë encouraged him to recognise the link between the rows and columns and assisted him to link his prior knowledge of groups to multiplication. Zoë's questions supported him in using those pieces of information together to efficiently calculate the area of the rectangle.

Excerpt 31

Sean was working out the area of several cut out shapes. He traced a rectangular shape on squared paper and went on to count all the squares.

Zoë *How did you count up all the area of the rectangle?*

Sean *I counted down all the sides and on the square there are bits on the line in the middle of the line.*

Zoë *Yes.*

Sean *So I count 2 of the bits as 1.*

Zoë *Did you count all the squares in the whole rectangle?*

Sean *Yeah.*

Zoë *Can you see any columns of the same size there?*

Sean *Yeah.*

Zoë *Can you show me where they are?*

Sean *There.*

Zoë *OK, next.*

Sean *There.*

Zoë *OK, so how many would you say there were?*

Sean *Ten.*

Zoë *Ten columns, are they the same? Just check.*

Sean *Seven.*

Zoë *Seven, can you see how many rows that are the same? Can you count the rows?*

Sean: *Ten.*

Zoë *Ten, yes. We will worry about this bit later. OK, so 7 columns and 10 rows. Can you use those numbers to work out how many squares altogether here?*

Sean *Each column is a 10.*

Zoë Ten that's right, so how many groups of 10 are there?

Sean Seven.

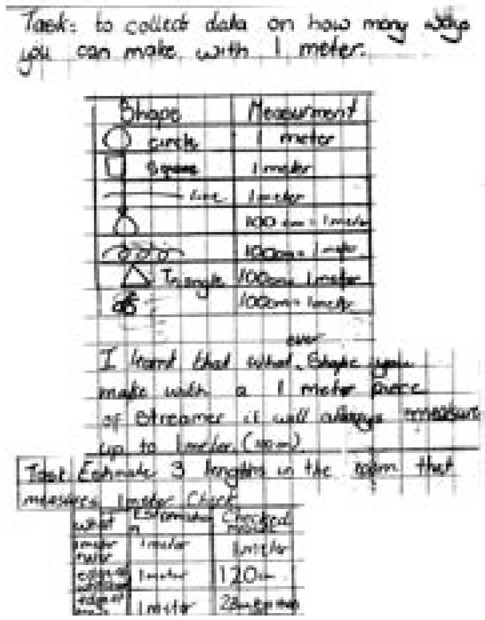
Zoë Seven groups of 10. How would you work that out quickly?

Sean Seventy.



The Growth Points therefore served as indicators of where students' conceptual understanding of measurement could be accessed through work samples, or orally through conferences. In this way Zoë was able to determine each student's needs and growth throughout the unit of work. This framework then assisted Zoë in deciding suitable tasks for individual students. After each session she collected the students' work and reflected on each work sample, as in Figure 25.

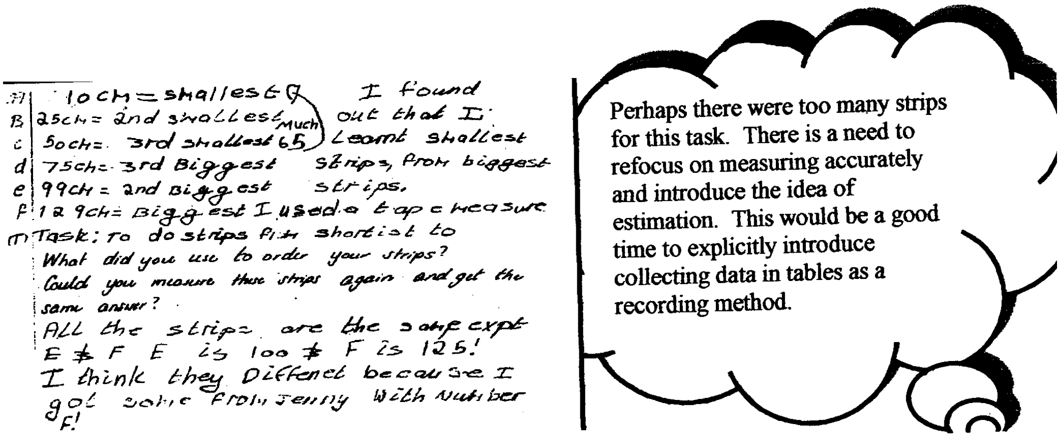
Figure 25. Excerpt from Zoë's case record



- Recorded her findings in a table
- Appears to have an understanding of conservation of length
- Applied the concept of perimeter in some of her examples
- Linked 1m to 100 cm
- Estimated and tested lengths in the room that would be close in length to 1 metre.

Zoë used the Growth Points to determine what she felt the student understood. She used these points to think through what she needed to do to further support the student, as shown in Figure 26.

Figure 26. Using the Growth Points to plan support



During a conference when Zoë worked with an individual student, she used the Growth Points to assist her when observing and questioning the student. These Growth Points provided a framework that allowed her to make sense of individual students' responses and further her understanding of their knowledge. In her case record Zoë referred to her use of the Growth Points:

> *A growing working knowledge of the Growth Points has helped me become more skilled in questioning to build on each learner's prior knowledge.*

(Zoë's case record)

In an interview at the end of the study, she commented that the Growth Points assisted her in thinking through the mathematics and it also gave her a new way to observe and respond to students. When interviewed about the usefulness of the Growth Points Zoë said:

> *It provided me with a new way to observe students' thinking and learning, it gave me a better way of knowing where next and what sort of questions to ask to achieve the desired end result. (Personal communication, 2001)*

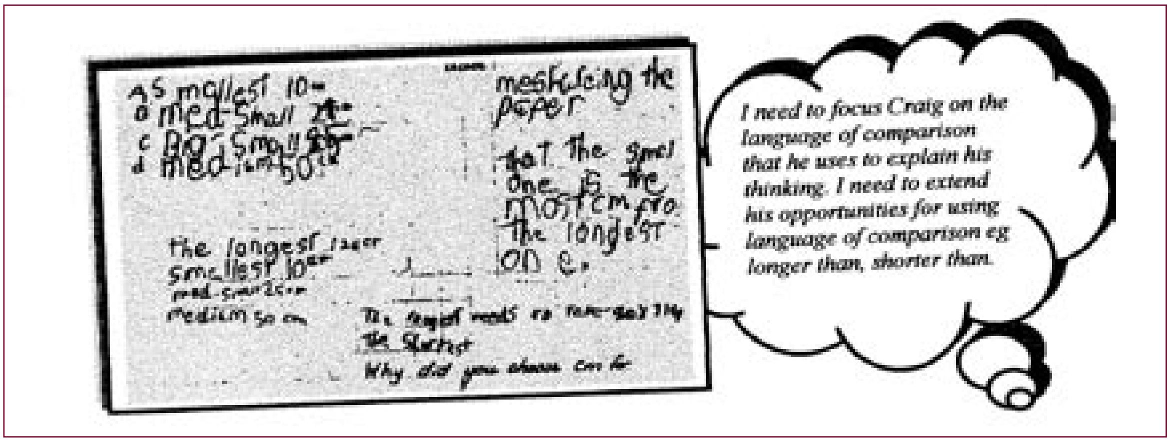
Designing investigative tasks

Zoë used the Growth Points when designing investigative tasks for her class. All students were set the same tasks but they did not necessarily go about them in the same way. The investigative tasks set allowed for multiple entries where students could bring in and use their prior knowledge. We observed that the tasks were generally open, though two instances were observed when the sorts of questions Zoë asked narrowed the tasks. Zoë also gave students conceptualised tasks at the end of each unit of work. The tasks required the students to apply the mathematics they had encountered in the classroom to another context. Usually the students worked collaboratively as they supported each other in bringing in the mathematics.

Supporting continual assessment

Zoë continually assessed the students and provided them with immediate feedback during all 3 components of the teaching process. She also gave them individualised written feedback in the workbook after each session. Zoë used the Growth Points as a way of supporting her judgements when responding and affirming the students' thinking and strategies (see Figure 27).

Figure 27. Using the Growth Points in assessment



Summary

The expectations Zoë had of her students were reflected in the strategies she employed in her classroom. The physical and social settings, the organisation of the students, the tasks, the materials she chose to expose her students to, the decisions she made about grouping and the classroom norms were responses to her beliefs about organising a classroom environment conducive to students' learning.

Zoë worked on the premise that talk/language is an important part of learning to know, hence the mathematics sessions had a strong language focus as Zoë attempted to build the students' oral and written communications. During the mathematics sessions Zoë encouraged the students to use their oral and informal language as she helped them to link their informal language to conventional mathematical language. Zoë emphasised the use of written communication as well. This required students to record their thinking in their own words. Possible recording strategies were often discussed during an introductory session, but during a conference with a particular student strategies were often discussed more thoroughly and if necessary Zoë assisted the student in thinking through more efficient ways of recording. Students were also exposed to other efficient strategies during a *sharing time* at the beginning or the end of a session. Communication as a means of building understanding and appropriating the mathematical language was a major focus.

The pattern of interactions described earlier indicates that Zoë primarily used one-to-one interaction strategies to support students on an individual basis in thinking through and building onto the mathematical concepts and in linking their informal/prior knowledge to the mathematical language. She used whole-class interactions when introducing the students to new concepts where she could discuss ideas and language creating a list of words that supported the students as they built their mathematical vocabulary and their understandings. She also used whole-class interactions to model efficient and useful strategies; however, conferencing individual students took most of the session time. Her questions required the students to put their thinking into words to communicate to the rest of the class. Students were also observed talking to themselves when asked a challenging question or when solving a challenging task. It appeared that students' confidence improved significantly. This could be either because students felt that their ideas and strategies were valued or because they received immediate feedback from the teacher and peers within a positive learning environment.

Zoë used the Growth Points as a conceptual framework for planning a unit of work, designing investigations/tasks and assessing students' thinking and understanding of the mathematical concepts. In an interview at the end of the research Zoë stated:

> *The Growth points gave me a new pair of eyes that I could use when observing students in my class, it provided me with deeper understandings of what individual students' needs are and how I could support them. It also supported my understanding of the mathematical concepts, the big ideas in measurement, and the key ideas in South Australian Curriculum Standards and Accountability Framework (SACSA).*

Case Study 3: Sylvia

Sylvia joined the teaching profession about 12 years prior to the research period. At the time of this study she was in her third year at St Luke's Primary School.

School setting

St Luke's Primary School is located in a suburb of Adelaide, South Australia. The school is co-educational with a population of approximately 330 students ranging from Reception to Year 7. In the year of the research, classes were predominantly composites. Sylvia's Year 5/6 class was in a transportable/demountable room next to 2 other Year 5/6 classes. Her class comprised 30 students of mixed ability and mixed gender, including 2 students with special needs. All students in the class did the same work throughout the year though the data analysed were of the 14 Year 5 students.

According to the Adelaide Social Atlas (2001) this suburb comprises a mixture of low to high-income families. The school reflects the economic spread in the suburb; school records indicate that students have parents who are classified as having low socio-economic status, while others have parents classified as high-income earners. Approximately 15 per cent of the students from low-income families are on school card (the measure commonly accepted in South Australian schools as an indicator of socio-economic disadvantage). One third of the school community is of Italian background, a very small number of the students have parents who are from the Philippines, and the remainder are of Anglo-Australian origin.

Physical setting of Sylvia's classroom

The physical layout of Sylvia's classroom was set up to assist students to work together in a relaxed and supportive environment as they investigate and learn to think mathematically. The physical setting has been divided into 3 main parts: the layout; resources: and the display. Each part seemed significant in assisting students as they made meaning of mathematics.

Layout

Sylvia's classroom was set up for small group work, with tables organised to allow groups of 6 to 8 students. There was a large area at the front of the class near the whiteboard where the class sat for whole-class discussions, usually at the beginning of a lesson. Students could also use the space in front of the class for small group work, if they wished, as some of the children seemed to prefer working on the floor.

Resources

At the beginning of all the sessions a range of materials, which could be used for investigations, were placed on the floor by Sylvia and some of the students. Students were free to pick any material they wished to use when undertaking investigations. If they wanted to use any other available materials, they were free to get them from the shelves. Resources were easy to access and clearly labelled.

Displays

Sylvia's class was bright with plenty of students' work on display. The mathematics/numeracy displays comprised individual students' work samples and work produced in collaborative activities amongst small groups of students. Also on display were lists of mathematical terms arising from whole-class discussions. The list stayed up for a whole term and new words were continually added. Sylvia encouraged the students to refer to the displays as a support to their learning. Displays were also used as a means of supporting the students when the teacher was not accessible. For example, students referred to the displayed list of terms when recording their thinking and findings. Students were also observed directing others to the list of words when assisting peers with their mathematical language.

According to Sylvia, displaying students' work samples was a way of valuing students' thinking and strategies. It was also a means of exposing students to other possible ways of solving mathematical problems or different starting points for investigating a mathematical concept or idea. In an attempt to ensure equity where all students' work was valued, Sylvia changed the display several times during the term making sure that each student's work was represented.

Social setting of Sylvia's classroom

Grouping strategies

At the beginning of the year Sylvia divided the students into mixed ability groups within their year levels. When questioned about her grouping strategies she expressed concern that mixing the groups might hinder Year 6 progress. As Sylvia reflected on the students' performances during the year, she commented that Year 5 students were performing better than Year 6. She therefore decided to mix the 2 year levels and as a result, Year 5 and Year 6 students worked together in mixed ability and mixed gender groups.

Students were free to choose whom they sat next to but they collaborated with other members of the group. Most students, however, tended to organise themselves in pairs and therefore collaborated with a particular person most of the time. There were some students who tended to work individually, although this did not happen all the time. Students often changed their mode of working throughout a unit of work. Sylvia changed the groups at the beginning of each term.

Protocols – encouraged behaviours in the classroom

Sylvia encouraged the students to interact by establishing class protocols to support the group-work setting. The class protocols required the students to ask for assistance from any member of the group before asking the teacher. All students were expected to engage with the task and to respect each other's ideas. Students were also encouraged to express their ideas within the small group or in the whole-class setting. Most students respected the protocols, although there were times when some students (in particular, Pedro) did not strictly adhere to the protocol and were often observed seeking Sylvia's attention.

Throughout the learning process students were expected to document their thinking and to refine their strategies and mathematical language. Before starting any investigations students were encouraged to discuss the task, ensuring that they all understood which data they needed to collect and what strategies they could use. Once the data was collected they had to analyse the data, make conjectures and draw conclusions. Students were expected to record their thinking in their own way.

Classroom interactions

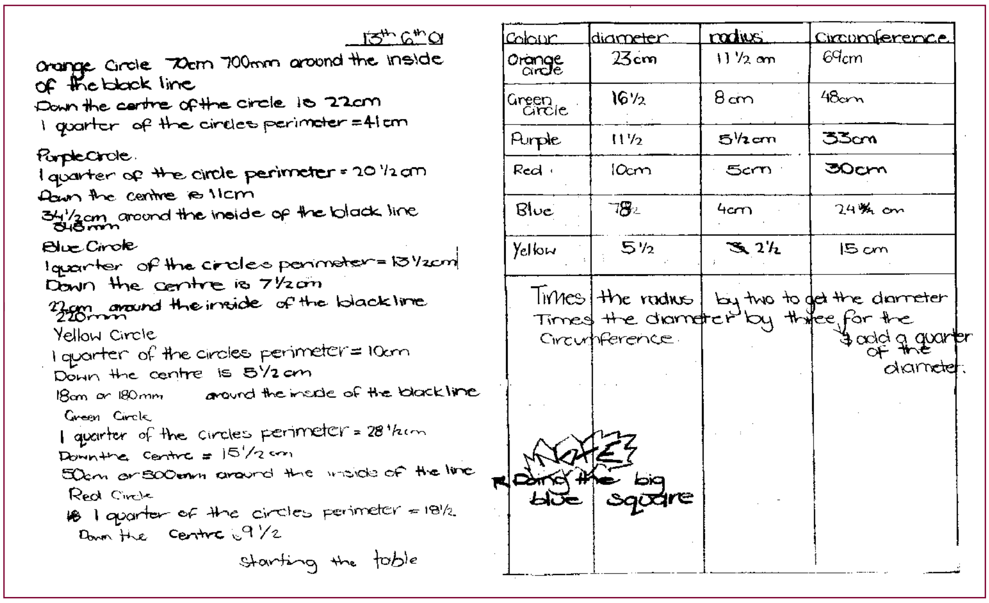
Data from 9 fortnightly observations show that Sylvia spent most of the lesson time interacting with students either on a one-to-one basis or with the whole class. Sylvia's classroom was organised to allow this high level of interaction to take place. For example, students were seated in mixed ability groups and the class protocols required the students to ask any member of the group if they were having difficulties before putting their name on the board to indicate to the teacher that they required assistance. We observed that the protocols that operated in the room and the strategies Sylvia put into place encouraged these interactions. Students were willing to support each other and they did not feel threatened when the teacher interacted with any one of them for a relatively long period of time during a session.

Students' working styles

The students were organised in mixed ability groups, and the protocols were set up so that students could support each other as they investigated mathematical concepts and solve mathematical problems. At the beginning of the year all students were given the same task. Later on, groups might have different tasks, although each student in a particular group would be working

on the same task. As the year progressed Sylvia's strategies of assigning tasks to groups changed and as a result students in the same group were not always investigating the same concept. For example, some students could be exploring area of rectangles while others in the same group could be determining the area of triangles. Also, even when students were investigating the same concept, they could be going about it in different ways. At other times, because students had already decided on strategies they wanted to employ to start the investigation (according to their prior knowledge), they were all given the same task. At these times, the direction the students followed depended on how Sylvia supported them as they built onto their prior knowledge. Figure 28 shows the work samples of 2 students who were in the same group and who were working co-independently. They discussed ideas and shared strategies but they recorded their thinking differently and it seems that they constructed a slightly different understanding from the same task.

Figure 28. Recording of the same task by two students who were working together



13th 6th 01

Orange Circle 70cm 700mm around the inside of the black line
Down the centre of the circle is 22cm
1 quarter of the circles perimeter = 41cm

Purple Circle.
1 quarter of the circle perimeter = 20 1/2 cm
Down the centre is 11cm
34 1/2cm 348mm around the inside of the black line

Blue Circle
1 quarter of the circles perimeter = 13 1/2cm
Down the centre is 7 1/2cm
22cm 220mm around the inside of the black line

Yellow Circle
1 quarter of the circles perimeter = 10cm
Down the centre is 5 1/2 cm
18cm or 180mm around the inside of the black line

Green Circle
1 quarter of the circles perimeter = 28 1/2cm
Down the centre = 15 1/2 cm
50cm or 500mm around the inside of the line

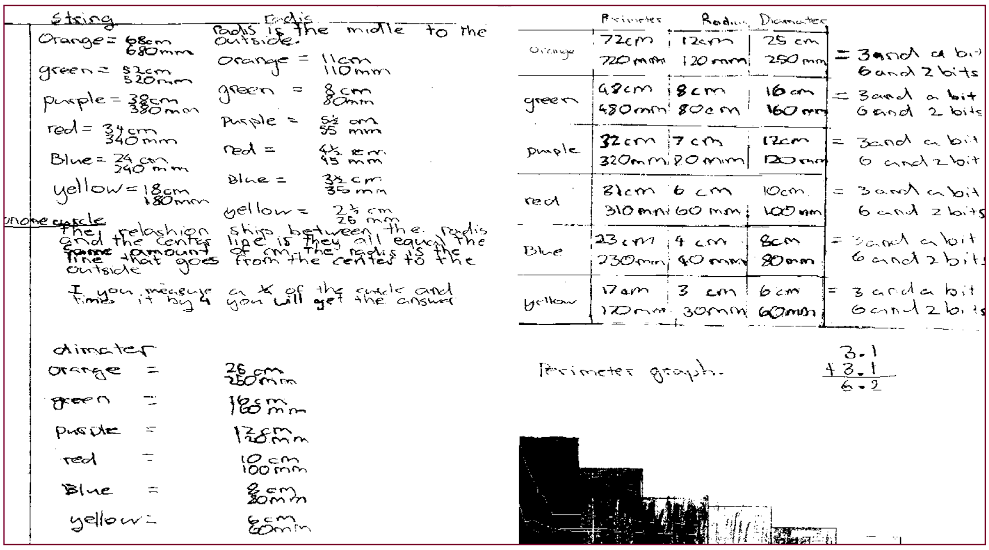
Red Circle
1 quarter of the circles perimeter = 18 1/2
Down the centre 9 1/2

starting the table

Colour	diameter	radius	Circumference
Orange circle	23cm	11 1/2 cm	69cm
Green Circle	16 1/2	8 cm	48cm
Purple	11 1/2	5 1/2 cm	33cm
Red	10cm	5cm	30cm
Blue	8	4cm	24 cm
Yellow	5 1/2	2 1/2	15 cm

Times the radius by two to get the diameter
Times the diameter by three for the Circumference
add a quarter of the diameter.

NOTE
Doing the big blue square



String
Orange = 68cm 680mm
green = 52cm 520mm
purple = 38cm 380mm
red = 34cm 340mm
Blue = 24 cm 240 mm
yellow = 18cm 180mm

Radis
Radis is the midle to the outside.
Orange = 11cm 110mm
green = 8 cm 80mm
Purple = 5 1/2 cm 55 mm
red = 4 1/2 cm 45 mm
Blue = 3 1/2 cm 35 mm
yellow = 2 1/2 cm 25 mm

one cycle
The relashion ship between the radis and the center line is they all equal the same amount of cm. The radis is the line that goes from the center to the outside

If you measure a 1/4 of the circle and times it by 4 you will get the answer

diameter
orange = 25 cm 250mm
green = 16cm 160 mm
purple = 12cm 120mm
red = 10 cm 100mm
Blue = 8 cm 80mm
yellow = 6cm 60mm

	Perimeter	Radius	Diameter	
Orange	72cm 720 mm	12cm 120 mm	25 cm 250 mm	= 3 and a bit 6 and 2 bits
green	48cm 480mm	8cm 80cm	16 cm 160 mm	= 3 and a bit 6 and 2 bits
purple	32cm 320mm	7 cm 70 mm	12cm 120 mm	= 3 and a bit 6 and 2 bit
red	31cm 310 mm	6 cm 60 mm	10cm 100 mm	= 3 and a bit 6 and 2 bits
Blue	23cm 230mm	4 cm 40 mm	8cm 80mm	= 3 and a bit 6 and 2 bits
yellow	17cm 170mm	3 cm 30mm	6 cm 60mm	= 3 and a bit 6 and 2 bits

Perimeter graph.

3.1
+3.1
6.2

The mathematical experience

The facilitation of the mathematical experiences observed was focused around two significant times during each session and the use of Growth Points as a conceptual framework. The two significant times were the *setting the scene and tone* and *Students' investigations time and conferencing*. Growth Points were used as a conceptual framework for planning units of work, mathematical investigations and assisting the teacher when assessing the mathematical experience. During the *setting the scene and tone* and *students' investigation time and conferencing*, Sylvia interacted with the students in groups as well as individually. The *setting the scene and tone* and the *students' investigations*, focusing particularly on conferencing, are described below. This will be followed by an analysis of the types of interactions observed in Sylvia's classroom, backed up by excerpts from observations and suggestions of their implications for students' thinking.

Setting the scene and tone

The *setting the scene and tone* always started with all the students at the front of the class sitting on the carpet in a semi-circle facing the teacher who was seated on a chair in front of them. During this time Sylvia involved the students in whole-class discussions about a mathematical concept, or brainstorming terms associated with the concept that they might find useful.

Sylvia used questions to establish the students' prior and informal knowledge of the concept. Once this knowledge was established, the questions that followed supported the students to build onto their prior and informal knowledge. Sylvia also supported the students in linking this knowledge to the concept being investigated.

The investigative tasks were presented orally to the whole class. Students also received the task in written form. Using a series of questions Sylvia engaged the students in discussing the meaning of the task and different strategies they could use in their investigation. Sylvia positively acknowledged all the strategies students offered. In certain cases she questioned the students further to build onto what was offered, however she tried not to indicate preference for any strategy. Discussions of possible strategies suggested to the students that there are many ways of entering and investigating the task.

When the beginning of the session was a continuation of an investigation that the students had been working on in previous lessons, Sylvia told the whole class what the task/s were, as there could be different investigations running alongside each other. She gave them the task in a written form and advised them where the materials they might need were. She answered any questions the students had regarding the investigations. Sylvia always reminded the students of the protocols and the need to record their thinking as well as data, conjectures and findings. The students then moved into their respective groups ready to start their investigations.

Students' investigations time and conferencing

During the students' investigations time Sylvia spent most of her time conferencing students. Between conferences she moved around the room to support students who required her assistance. When Sylvia selected a student for conferencing, she began by asking the student to explain their thinking.

This pattern was consistent in all of her conferences. Because the student had to verbalise their thinking, Sylvia was able to gain access to the student's reasoning at the time. There were instances, however, when the student's

explanation was too brief or did not give insight into current reasoning. In this case Sylvia asked the student further questions of the same type as those previously asked. Such questions have been categorised in the analysis as 'establishing questions'.

Towards the end of the year we observed that students started to verbalise their current reasoning to Sylvia as soon as she came near them for a conference. Students were, towards the end of this research, asking similar questions of their peers when their peers needed assistance, so instead of just providing answers to questions they were asking questions of their peers.

A conference always started with a question where Sylvia tried to discover the student's thinking. This could be on the strategies the student was using and/or the mathematical ideas. The student explained their thinking to Sylvia who then proceeded to further question the student. The questions she asked supported the students in making the link between prior knowledge and that being investigated. If the student had already made the link, Sylvia's questions were then either to further challenge the student or to build on existing knowledge. Sylvia also used questions to support students in the development of a mathematical way of thinking, referred to here as the 'mathematical process'.

During a conference, if the students had difficulties making links to previous knowledge or when asked a challenging question, Sylvia felt that if the student needed more thinking time, she left them with a question and moved around the classroom answering questions from other students who needed assistance. Sylvia returned to the student to complete the conference a few minutes later. This allowed the student time to think through the idea and to try to arrive at a solution. If the students could not call on prior knowledge, and if this bit of knowledge was imperative to the understanding, she explicitly taught the student. She could later give the student different exercises to reinforce the concept. Students were, therefore, often given smaller investigations or received individually explicit instructions when working on a larger investigation.

Conferencing of individual students formed the core of all the lesson time. Normally a mathematics session lasted 60 minutes. During this time Sylvia averaged 5 10-minute conferences. Conferences were initiated either by Sylvia or by the students. The data shows that the students initiated 14 per cent of the conferences in all the sessions observed. Since Sylvia initiated 86 per cent of the conferences she was therefore controlling which students she worked with on a one-to-one basis. She kept a record of the students she conferenced each session to make sure that over a set period of time she had worked with the whole class of students on a one-to-one basis hence ensuring equitable access to teacher time.

Excerpt 32 illustrates a conference between Sylvia and Gabby.

Excerpt 32

Gabby had been collecting data using a range of strips cut to different lengths, including fractions of a metre. All lengths were cut in multiples of 10 centimetres. Gabby had measured and recorded the measures in centimetres, except the one metre only measure.

Sylvia *Could you tell me what you have done so far?*

Gabby *I have measured these strips and recorded the measures.*

Sylvia *I notice that you have recorded all your data in centimetres. Is there another unit you could record them in? For example, how would your numbers change if you recorded them in metres?*

Gabby *It's still 6 cm.*

Sylvia *So how would you record that in metres?*

Gabby *But it is not a whole metre.*

Sylvia left Gabby to think about it …

Gabby *[five minutes later Gabby called to Sylvia] I know 25 cm is a quarter of a metre.*

Sylvia *What do you think this strip might be? [pointing to the 80 cm strip next to the metre ruler]*

Gabby *[measuring the strip] It's 80 .Oh! Over 100.*

Sylvia left Gabby to continue with her investigation with the intention of taking Gabby's thinking onto decimal fractions in the future.

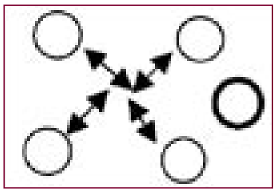
Sylvia challenged Gabby to convert her measurements to metres. When Gabby experienced difficulties converting the different centimetre measurements because her recording measurements did not include metres Sylvia left her to reflect on how she could use her prior knowledge. Gabby was using 25 centimetres as a benchmark, as she already seemed to be familiar with the fractional relationship between the two units. By giving Gabby time to think, Tania allowed her to draw on her existing understanding of centimetres and metres, though at this stage she was unable to single-handedly convert other measurements into metres. Sylvia's next set of questions supported Gabby in making the connections and in working out the relationships. Sylvia's questions encouraged Gabby to recall and use her prior and informal knowledge, linking it with formal terminology, thereby eliciting mathematical thinking.

Throughout the conferences Sylvia used questions to generate discussions with the students. She affirmed their responses and gave them immediate feedback as well as immediate challenges, which seemed to increase student participation and increase their confidence.

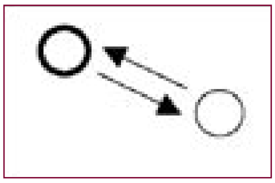
Interpretive summary of classroom interactions

The systematic analysis and interpretations of the observational data resulted in identifying a number of different modes of interactions between the teacher and students and amongst the students themselves. Detailed are the four categories of interactions and excerpts from classroom observations and work samples. Tentative statements about the significance of the social setting to advancing mathematical thinking have been made. The interaction types identified were:

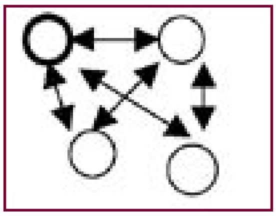
Category 1 Small group interaction during investigations, with or without the teacher observing



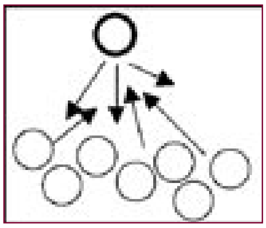
Category 2 One-to-one interaction (conference) between a student and the teacher



Category 4 Small group or pair of students with the teacher interacting and observing at different times



Category 5 Whole-class interaction with the teacher occurring during Introduction times



The dynamics of Sylvia's mathematics sessions involved students and teacher in a range of purposeful interactions, where the students worked with the teacher or in small groups exploring mathematical concepts and building mathematical understandings. In Sylvia's classroom some of these interactions, for example, category 1 and 2 occurred concurrently. Sylvia instigated mostly category 2 and 4 interactions. Sylvia initiated most of category 2 interactions though the students initiated about one fifth of such interaction. Described below are the different categories of interactions observed in Sylvia's classroom.

Category 1 Small group interaction, occurring within student investigations

Using plastic grids 2 boys were finding the area of triangles by counting squares. While they had the same task and chose to use the same strategies they were each investigating their own triangular shape by counting the centimetre squares. Excerpt 33 is a short episode of the interaction between the 2 boys to show how they supported each other in working out the fractional parts.

Excerpt 33

George	*Area must be …*
Alan	*OK, you have a quarter there and a quarter there that makes a half. What are you talking about?*
George	*Oh, I know there is $\frac{3}{4}$ there and $\frac{1}{4}$ there, that makes 1. I just have to continue … I think I will get it.*
Alan	*What is the area of this shape?*
George	*This one here is $36\frac{1}{2}$*
Alan	*$36\frac{1}{2}$?*

George started to erase his work.

Alan	*No. You've got to find out why you have got this; you need to figure it out.*

Alan assisted George in realising that he was not counting all the space occupied by the shape – he was leaving out the fractional parts. Alan encouraged him to reflect on why he obtained the answer he did and to move from there. George actually stopped erasing and proceeded to rethink.

Category 2 One-to-one interaction (conference) between a student and the teacher

Excerpt 34 is part of a conference between Sylvia and Pedro. Sylvia's questions demanded that Pedro explain his thinking. Pedro was measuring the width and height of rectangles and working out their area. He was trying to find out the relationship between the linear dimensions and the area of the rectangles. When Sylvia challenged Pedro to explain his strategies; she found out that Pedro was measuring the linear measurements and multiplying them together. He was experiencing difficulties with fractional parts, as he did not know how to multiply decimal fractions. Sylvia's questions supported Pedro in connecting with and extending his prior knowledge of area.

Excerpt 34

Pedro *Times this.*

Sylvia *Why do you times this?*

Pedro *Because …*

Pedro picked up his calculator and entered the numbers while the teacher waited.

Pedro *How do you do a half?*

Sylvia *OK, the half on the calculator is what?*

Pedro *0.5. … [Pedro returned to the calculator]*

Sylvia *Because if this in here is .5 [pointing to the half squares] and .5, altogether they are whole.*

Pedro *OK.*

Sylvia *So can you link these all up [demonstrating the linking of two half squares]*

Pedro *It's 9 and ½ going down, if I times 9 by 10.5 all those will get filled up.*

Sylvia *Exactly. So this is a quick way of adding those squares up isn't it? [Pedro nodded.]*

Sylvia *How did you work that out? Let me see. Now what are they? What is 94.5 actually?*

Pedro *They are square centimetres.*

Sylvia *Why are they considered as square centimetres?*

Pedro *Because they are the same size.*

Sylvia *Yes, but what are we measuring?*

Pedro *Centimetres.*

Sylvia *Are these centimetres? When I think of a centimetre I usually think of a ruler, and square centimetres I think of grid paper. So why the square?*

Pedro *Because there is a centimetres going up and down [pointing to the height and width of the square] There is centimetres all around.*

Sylvia *[Completing the phrase for him] Making square centimetres.*

Pedro's responses to Sylvia's questions gave Sylvia an understanding of his thinking. It also provided her with an insight into Pedro's prior knowledge of fractions, area measurement and standard units for measuring area. Having insights into Pedro's understanding she was then able to support him in linking and building on this knowledge to understand relationships between linear measurements and area. She assisted him to understand what the measurements actually meant and what multiplication is actually doing. The one-to-one conferencing gave Sylvia the opportunity to support Pedro's individual needs in a non-threatening environment.

Category 4 Small group with the teacher interacting and observing at different times

Excerpt 35 is part of a discussion between Dahna, Louise and Sylvia. Louise and Dahna worked together calculating the area of a triangle. The interaction shows Dahna taking a more active role than Louise in the conversation.

Excerpt 35

Both students had copied the triangle into their books.

Dahna *[to Louise] I think you could times the 2 sides together and then divide it by 2.*

Dahna *[to Sylvia] We've measured the 2 sides.*

Louise *She should write down what the 2 sides are here.*

Sylvia *What are you trying to do now what do you think it is?*

Dahna *We measured those [sides] and then timesed these 2 together, if we times this together we thought it would come up with the area of another shape … the area of this shape [quadrilateral].*

Sylvia *Why divide by 2?*

Dahna *Because that will take away the other half.*

… … …

Sylvia *What did you do?*

Dahna *I counted the full square and I got these, it was too hard to match up because none of these really ... you could not tell whether they are right or not so you always come up with dodgy answers. Like I matched up those two halves together, this one is a little bit and these are bigger.*

Louise *And this one.*

Although Sylvia directed the questions to both students, Dahna dominated these interactions. However, Sylvia did not try to involve Louise more. In such interactions both participants were active though it seems that one took a more passive role when the teacher was present. As soon as the teacher left, Louise became more talkative and was making significant suggestions to solve the problem. It is an important aspect of classroom interactions (see, for example, Hoyles, 1985) as both students engaged with the task.

Category 5 Whole-class interaction with the teacher occurring during introduction times

All sessions observed started with whole-group interactions, classified here as Category 5. At the beginning of the session Sylvia set the scene and tone either by linking back to an investigation that the students had already commenced or by introducing a new concept. The length of the interaction depended on whether it was a continuation of an investigation or the introduction of a concept. Sylvia questioned the group as a whole and any member of the group who volunteered was invited to respond. The students also asked Sylvia questions that she either answered or redirected to the class. It was observed that Sylvia asked and answered most questions during this part of the interaction.

Excerpt 36 is from a whole-class interaction. The students had worked on perimeters of shapes in previous lessons. Using a series of questions Sylvia ascertained their prior understanding of perimeter before she sets out to extend this with a new understanding.

Excerpt 36

Sylvia Today you will all be doing the same activity; it is already set up on your tables. We are going to look at perimeters of circles. What is the perimeter? Do you remember we looked at the perimeters of the rectangles in our school? What is perimeter?

Student It is the outside of a shape.

Sylvia Yes, it was on the outside of the shape.

Sylvia presents the task.

Sylvia Find a relationship between the perimeter of any circle and its radius and diameter.

She reminds the students of the need to collect data and to find a pattern.

Sylvia Is there a relationship between the diameter and the radius?

Student They can be drawn on a circle.

In this type of interaction Sylvia's questions supported the students in linking the current mathematical activity to the concept of perimeter investigated in previous lessons. Students' questions were mostly for clarifying purposes and to link previously learnt concepts to the task.

Table 8 summarises the types of interactions in which Sylvia engaged during the observed lessons. The pattern of interactions shows that Sylvia spent 72 per cent of the mathematical investigation time working with students on a one-to-one interaction, which seems to be her preferred mode of interaction. The protocols Sylvia had in place supported the one-to-one interaction, allowing her quality time where she could work closely with individual students.



This mode of interaction allowed her to access (in a non-threatening environment) and know how and when to challenge and support the students thinking in the meaning making process. While Sylvia worked with individual students, the rest of the class either worked collaboratively or co-independently. They were able to support each other, freeing Sylvia to focus on individual students' needs and growth.

Table 8. Summary of Sylvia's interactions

Categories of interaction	Introduction	Student investigation
Category 2 One-to-one interaction between a student and the teacher	-	72%
Category 4 Teacher with a small group or pair of students	-	28%
Category 5 Whole-class interaction with the teacher occurring during introduction times	100%	-

Sessions to introduce new concepts averaged 15 minutes but if the students were to continue with an investigation Sylvia briefly told the students what to do, which lasted about 3 to 5 minutes. Sylvia reminded the students of possible terms/concepts that might be useful, access to materials, methods of collecting, organising and analysing data and the class protocols. Whole-group interaction time therefore varied from 3 minutes to 15 minutes.

During a whole group interaction Sylvia used questions to ascertain the students' prior knowledge and again her questions supported the students in creating links with prior knowledge and in building new knowledge. Sylvia orchestrated all whole-group discussions. Students were encouraged to ask questions, although most of the questions were asked by the teacher. Students' questions were either answered by Sylvia or redirected to the whole group, leaving Sylvia in control of the discussions.

All students were expected to record their thinking and understanding in their own way even when they discussed and shared ideas together. This gave Sylvia further access to individual students' thinking and understanding since she used the students' work samples as part of their assessment.

Analysing the types of questions Sylvia used during a conference

Student-student interactions or teacher-student interactions (as described in the section Classroom Interactions) were major elements of Sylvia's mathematics sessions. In all of the mathematics sessions observed Sylvia used questions to generate discussions with the whole class, small groups

of students, or individual students. The questions Sylvia used during those sessions have been systematically analysed and categorised according to what they purport to do in relation to students' thinking and learning. Questions were analysed under the following categories:

- Reflecting and explaining current reasoning
- Refining and building onto mathematical thinking and strategies
- Supporting conceptualisation of the mathematical process.

Reflecting and explaining current reasoning

Questions classified in this category required the students to justify and explain their current reasoning and understandings. These questions provided Sylvia with insights into individual students' thinking and understanding of the mathematical ideas being investigated. In Excerpt 37, Sylvia asked Pedro to justify his reasoning.

Excerpt 37

Pedro	*I multiplied by 10.*
Sylvia	*Why 10?*
	… … …
Sylvia	*You have recorded this as a fraction. Why did you do this?*
	… … …
Sylvia	*Why have you put them over 10 now?*
Sylvia	*You have put 10 there for a reason, why the 10?*

(From observation 1)

In each of the above questions Sylvia was attempting to get Pedro to justify the use of 10. It seems that Sylvia was aware that Pedro was applying his prior knowledge, but she wanted to be sure that he understood the rule he was applying and its relevance to the mathematical idea being investigated. In another conference, Sylvia's positive response (see Excerpt 38) to John's explanation was followed immediately by a further question requiring him to justify his thinking.

Excerpt 38

Sylvia	*Yes, you are right – why does it work?*

(From observation 1)

By verbalising his thinking John had to reflect on the mathematical ideas and strategies he had applied.

During an investigation of the area of triangles, Gabby and Lynne used their knowledge of the area of a rectangle to work out the area of the triangle of equal base and height. They conjectured that dividing the area of the rectangle by two would give them the area of the triangle. Sylvia's question, *Why divide by two*? from observation 6 required the students to go back to their previous thinking and to come up with logical arguments.

Questions that required the students to explain their thinking and the strategies they were using were often asked in the initial part of a conference or during a conference. Sylvia used these questions as a means of establishing students' current reasoning which she used to inform her next question. Excerpt 39 provides examples of this kind of questions:

Excerpt 39

Sylvia	*Show me how you worked that out? [pointing to the base and height]*
Sylvia	*So what is 3.5 representing? What is it showing?*
Sylvia	*What are you looking for?*

(From observation 4)

Sylvia	*When you did this, what did you do?*

(From observation 3)

Each of the above questions required the students to explain their current reasoning and understanding of the mathematical concept and strategies. Having to provide an explanation or a justification engaged the students in reflecting on the mathematical ideas and strategies, hence making them conscious of the mathematical knowledge they were learning or investigating and how they relate to other ideas (reflection).

Refining and building onto mathematical thinking and strategies.

During a conference Sylvia supported Gabby in recording her measurements in metres as opposed to metres and centimetres. Sylvia used a strip of paper to demonstrate to Gabby the links between her measurements and how they could be thought of as fractional parts of a metre (see Excerpt 40).

Excerpt 40

What would happen if you fold it in half?

(From observation 1)

Can you show me how you would do that using some of the equipment?

(From observation 5)

Do you think it would make a difference if you used the actual measure rather than the rounding off number?

(From observation 7)

Sylvia used questions to encourage students to refine their strategies and to build on to their existing knowledge. Her use of hypothetical questions was noted in a few observations. Such questions either alerted the students to a problem or they were used to get the student to look at different possibilities, which in the end served as a means of helping the student to refine mathematical understandings and strategies.

Supporting conceptualisation of the mathematical process

Questions were also used to support the students in thinking through the mathematical process, that is, the collection of data, organising data, and analysing data prior to arriving at a solution. Questions pertaining to this category did not have any set pattern; the questions depended on Sylvia's purpose in asking the question. Excerpt 41 shows typical examples of questions that Sylvia used when supporting students in making sense of their data.

Excerpt 41

What types of information have you collected?

So how many pieces of information are there?

So how many columns and what would the headings be of your columns?

Could you now draw your table up and see if it makes your data collection easier and your search for a pattern?

Can you notice a pattern or relationship in your data?

Do you think a table might help to organise your data so you can more easily search and identify a pattern?

These questions seemed to have supported students in thinking through their data, tabulating and organising their data and facilitating pattern searching in an attempt to build mathematical understanding and logical mathematical reasoning.

Though Sylvia's questions have been categorised into 3 different sets, the questions complemented each other, forming a framework that scaffolded students as they explored different mathematical concepts and linked their prior mathematical knowledge to current investigations. The questions gave Sylvia insights into the students' current thinking, thus supporting her facilitation of their mathematical experience. Having to verbalise their thinking, students became more aware of their own knowledge and understandings therefore explaining or justifying thinking or strategies. The hypothetical questions were more of a challenge but without confrontation. Typical

of these questions was: *If this is the case, then what about that?* or *Will this change?* The questions served to bridge the gap between what the student knew and understood, and what the teacher wanted this student to learn from the experience.

Using the Growth Points

Data on Sylvia's use of Growth Points was collected through classroom observations, her case studies and students' workbooks. As did all the teachers participating in this research, Sylvia used the pre-test data and the set of Growth Points to establish the range of thinking in her class, paying particular attention to individual students. This informed her of the students' prior knowledge in measurement.

Planning units of work

When planning a unit of work, Sylvia used the Growth Points as a conceptual framework. In her case record, under *mathematical ideas*, Sylvia listed some Growth Points she wanted the students to understand from the unit of work. Rather than copying the Growth Points as they were listed in existing documentation, she rewrote them to fit in with the mathematical ideas she wanted to teach.

Figure 29. Sylvia's use of Growth Points

Mathematical ideas

Area is the covering of two-dimensional or three-dimensional shapes

Area can be measured

The size of the unit will affect the measure

Standard units are needed to compare measure

Different shapes of the same size will have the same area- conservation

Different aspects of measurement – base, height, length, width

Formulae can be used to find area. (From Sylvia's case record)

Sylvia used Growth Points to assist her to think through possible questions she could ask to support individual students as they tried to make meaning of mathematical concepts during an investigation. She did not always ask the questions exactly as she wrote them in her planning framework and not all students were asked the same questions. Sylvia used the written questions as a framework to ensure that students were challenged to think through the unifying ideas of measurement and broader and more general ideas related to the SACSA. The questions also prepared her to see the different pathways students might take as they constructed and made meaning of the mathematics.

Designing investigative tasks

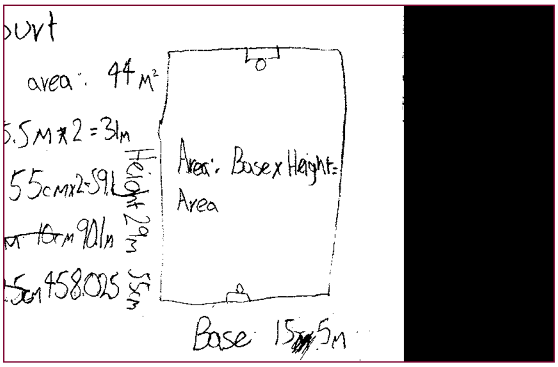
Sylvia also used the Growth Points when preparing mathematical investigations/tasks. The investigative tasks she designed were multi-entry; students could enter the task at different points using their prior knowledge and understanding. The open nature of the tasks was an attempt to provide challenges and opportunities for all students to identify with the demands of the task and to start using their own prior knowledge.

In order to promote links between the students' school mathematics and their numeracy development, some of the investigation tasks that Sylvia devised were contextualised tasks. Although they were mostly contrived tasks, they served to expose the students to different ways of thinking. As some of these tasks depended on factors other than mathematics, the students learnt that they had to make decisions about when to use particular information they might have. For example, in a contextualised task in linear measurement the students had to measure a netball court and a basketball court. Some of the students rounded their measurements to the nearest metre, applying the rule: *Round down when the number to the right of the decimal point is less than 5.* When questioned, they initially did not understand that this was inappropriate as they were 'correctly' applying

their prior knowledge. Using questions, Sylvia led them to reflect on their measurements and answer questions about the 'reasonableness' of measurements – *When should they round off? What does 0.5 of a metre look like?* and *Why is it inappropriate to round off if building a netball court?*

For example, in Figure 30, Sergio measured the rectangular border in metres and centimetres. He initially rounded down the base measurement from 15.5 m to 15 m but chose to leave the height at 29 m 55 cm. Sergio then applied his knowledge of the formula, calculating and recording the area as 443 m 25 cm. (this was a correct calculation for his rounded base measurement, although he did not have the correct area units). He corrected the crossed-out answers after the conversation documented in Excerpt 42.

Figure 30. Work Sample: Sergio



Excerpt 42

Sylvia	*How did you work out the area of the playground?*
Sergio	*I measured, then I rounded the numbers, and used the height times the base because it is a rectangle.*
Sylvia	*Do you think it would make a difference if you used the actual measure rather than rounding the numbers?*
Sergio	*I am not sure.*
Sylvia	*Why don't you have a try?*
Sergio then recalculated	*Wow! I was 14 almost 15 square metres out. That's a lot!*
Sylvia	*Why do you think you were out by that amount?*
Sergio	*Because I've left off .5 of a metre from the length which means I have left off a whole row of parts of squares. When put together it must be nearly 15 squares.*

The contextual nature of the task and the questioning helped Sergio to make sense of the measurements.

Sylvia planned to provide a contextualised task at the end of the 2 units to establish if the students knew which mathematical ideas to choose to solve the problem. She facilitated student learning through questioning and by encouraging students to reflect on their mathematical thinking and knowledge. Questioning was also used as a means of getting students to express their ideas and to link their language to the conventional mathematical language. The students also supported each other throughout the task.

Throughout the learning process Sylvia continually assessed her students through observation, analysis of work samples and conferencing. This assessment provided her and the students with ongoing and immediate feedback, which seemed to affirm or challenge student's understanding. Sylvia used the measurement 'G*rowth Points*' as a framework

to establish individual students' understanding in order to decide what directions to take. Sylvia's questions, therefore, were in anticipation of what she would find students doing during an investigation; questions she could ask to compel them to reflect on the concepts being investigated and the unifying ideas of measurements.

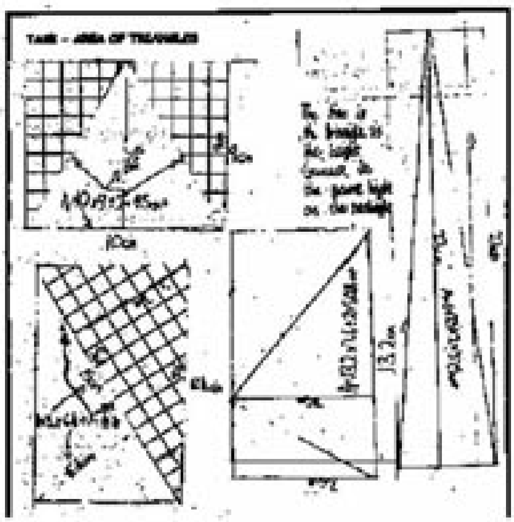
Supporting continual assessment

Sylvia continually assessed the students through conferences and their work samples. The Growth Points were used to describe the students' knowledge at different stages of the investigations and in the unit work, that is, they formed part of an ongoing assessment of her students. Figure 31 is an excerpt from Sylvia's case record to illustrate her assessment and reflection on an individual student. Sylvia was supporting Sergio in using a known fact (the areas of rectangles) to work out an unknown fact (the areas of triangles), and to eventually work out the formula for calculating the areas of triangles. Her reflection and assessment of his work indicated that he was developing an understanding of using rectangles to work out the area of more complex shapes, but he needed further experience.

Initially Sergio used plastic grid sheets and placed them over the triangle and counted the squares to find the area.

I then instructed Sergio to enclose the triangle in a rectangle and to find the area of the rectangle. He immediately noticed that the area of the triangle was half that of the rectangle. I then asked Sergio if this worked for all triangles and if so could he come up with a formula or rule to apply to the finding of the area.

Figure 31. Excerpt from Sylvia's case record



Sergio is able to identify the height of a triangle and once it is enclosed in a rectangle is able to work out the area by timesing the base by the height and then dividing by two however, I believe he is just at the beginning of understanding this concept.

Sylvia's questions required the students to verbalise their thinking. Sylvia then used the Growth Points as indicators against which she assessed the students' knowledge and as a conceptual framework to support her in phrasing subsequent questions. Sylvia also acknowledged that the Growth Points supported her when planning the unit of work, giving her a sense of direction and providing a framework when planning investigations and when assessing individual students' understanding and growth. The reflective nature of the study, and the use of Growth Points, forced Sylvia to focus on individual students' needs and growth. She felt that she was now better able to discuss individual students' growth than she was at the beginning of the project.

In the session described below, a range of different investigations was taking place simultaneously in the room. The focus here is on two pairs out of a group of seven students who were investigating the area of triangles. The task allowed the students to enter at their individual points of mathematical growth and build upon prior knowledge. Excerpt 43 illustrates what these two pairs did and how the teacher scaffolded learning for each of the students. The work samples also show that even when students were in the same group and working in a supportive way they shared ideas but followed through their own thinking in their recordings.

Excerpt 43

Nadia and Cathy decided that to work out the area of the triangle they needed to measure the 2 (slanting) sides of the triangle and multiply them together, then divide by 2. Sylvia overheard the girls talking to each other as they carried out the idea and asked: How will that give you the area of the triangle? Sylvia then left them to investigate. Meanwhile they calculated the area using their idea and attempted to verify their idea by tracing the triangle on centimetre grid paper and counting squares. Sylvia returned and questioned the girls:

Sylvia *What have you done?*

Nadia *I thought about the rectangles, the base and the height but I didn't think it's right so I didn't do it. … Then I measured those 2 and times together and divided by 2 and it didn't work. When adding up all those it was 92 and half.*

The girls thought about building on to their recently explored knowledge of rectangles, but decided against this idea. When their idea of using the two slanting sides of the triangle failed they went back to using their knowledge of the rectangle formula.

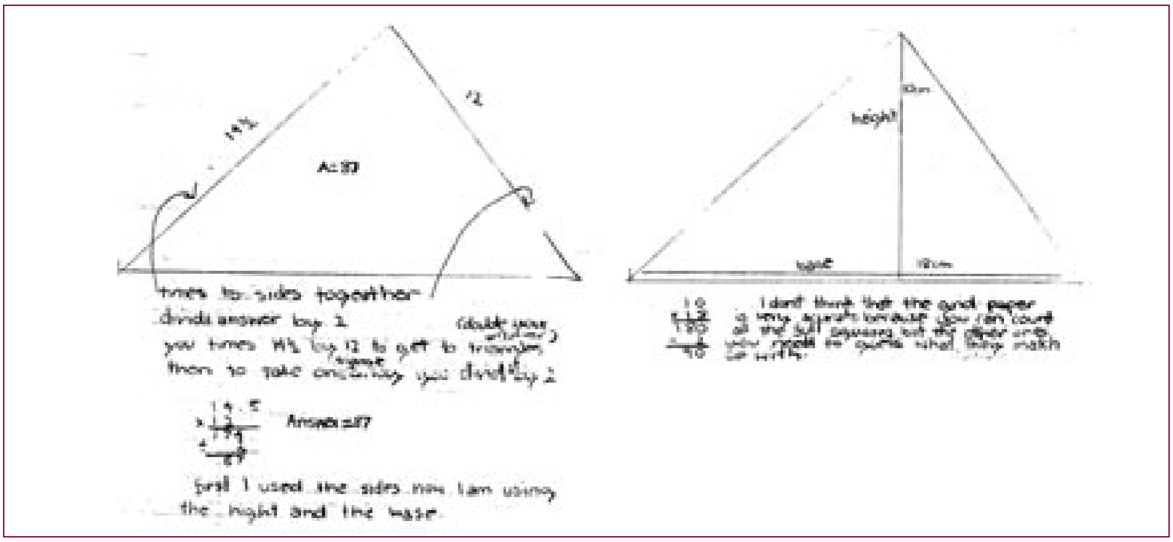
Nadia *Now we are doing the height and the base.*

Sylvia *What made you think of the height and the base?*

Nadia *Because that is what we did on the rectangles …*

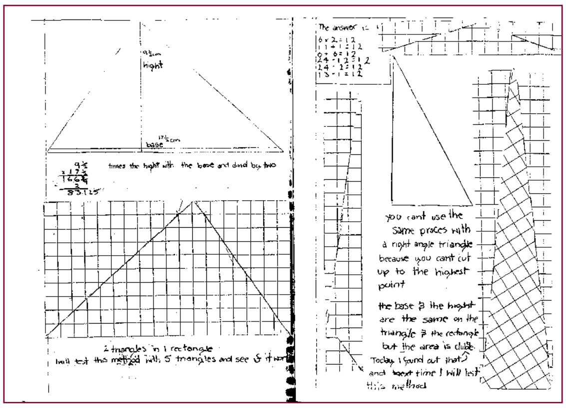
Nadia and Cathy hypothesised that their idea of base times height and dividing it by two would work for the triangle (see Figure 32). The next day they continued the investigation using other triangles and tested their conjectures.

Figure 32. Nadia's work sample from the first session when she multiplied the two slanting sides



From the work-samples in Figure 33 and observations it appears that Nadia is forming an understanding of the relationships between rectangle and triangle formulae, but more importantly she is developing a way of working and thinking mathematically which we hope will support her to become more numerate.

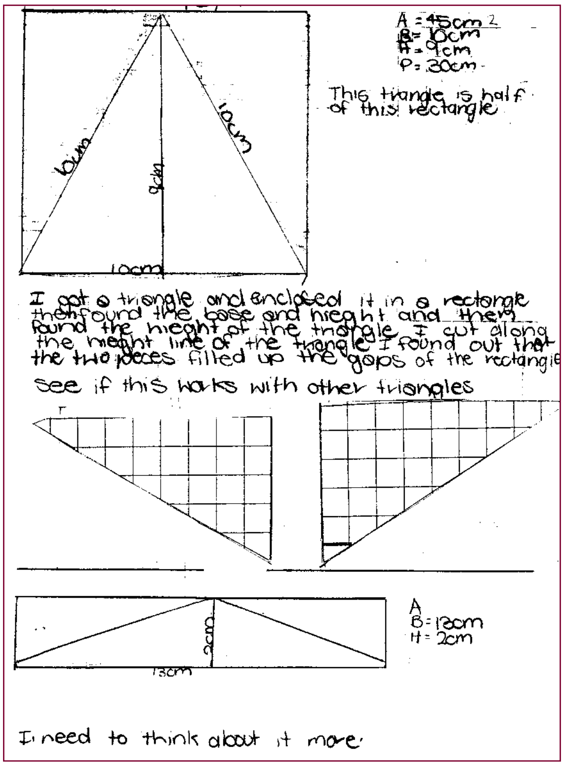
Figure 33. Work samples from the following sessions



While investigating triangles, Amelia and Sarah began by tracing the triangles onto centimetre grid paper and individually counting the units. They then decided to multiply 2 sides together ($7 \times 8 = 56$). Sarah commented: *That's how many are in the centre.* However, their calculations did not equal the area of the triangle. After observing and analysing their data Sylvia decided to redirect them. In the following session Sylvia altered the task. Amelia and Sarah were asked to explore the relationship between the area of the triangle and that of the rectangle with equal base and height. Following is a selection of Amelia's work samples. Figures 34 and 35 illustrate Amelia's investigation of the area of triangles during the second and third sessions.

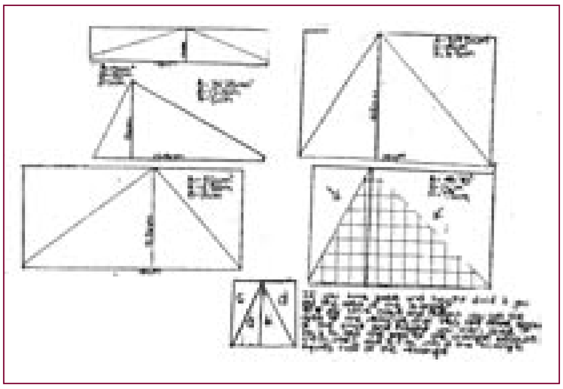
Amelia explored the relationship between the areas of a rectangle and triangle. When she drew triangles within a rectangle so that they had the same height and base as the rectangle (see Figure 34), she found that the area of each triangle was equal to half of the area of the rectangle. Amelia's written comment, *I need to think about it more*, suggests that this student sees the importance of reflection when investigating, and illustrates why Sylvia provides the time for this to occur.

Figure 34. Work sample: Amelia (1)



In the third session (Figure 35) Amelia continued her investigation using several other triangles and generalised the relationship between the area of triangles, their heights and bases. This generalisation was later expressed algebraically and recorded as $B \times H \div 2 = A$.

Figure 35. Work sample: Amelia (2)



These 2 examples demonstrate how Sylvia designed and facilitated the task to allow students to enter from their own stage of growth, build upon prior knowledge, develop mathematical thinking including making conjectures, and collect data to verify or modify their thinking.

Summary

The physical organisation of the room and the social setting, with mixed ability groupings, enabled students to work together as they constructed their own understanding. Classroom protocols were set up with the intent of supporting the sharing of knowledge and strategies amongst the students, and also allowing the teacher the opportunity to work with students on an individual basis. Displays were used to encourage or celebrate the use of different strategies the students had used, and to provide students with supports for linking, acquiring and using conventional mathematical language. Multi-entry tasks enabled all students to access the same task and to bring in prior knowledge. By moving in and out of a contextualised task students were given opportunities to apply their knowledge to real life situations.

The social construction of knowledge was done either within a small group setting where students supported each other, in a larger group (the whole class) during the introduction of mathematical concepts, or when Sylvia worked with individuals or pairs. Sylvia spent most of the mathematics investigation time conferencing individual students. By using a series of questions during conferencing, she determined each student's understanding of the mathematical concepts as well as the strategies this particular student chose to use. Sylvia was then able to support the students in linking prior knowledge to conventional mathematical knowledge and in building onto their current knowledge. Her questions challenged the students and acted as scaffolding, enabling the students to create links between the new knowledge and their existing knowledge and facilitating their choice of which knowledge to use when solving problems or what questions to ask of themselves and of each other.

The analysis of the questions Sylvia used during a conference and during the *setting the scene and tone* of the lesson show that the purpose of these questions was to support students in explaining their reasoning and thinking. Because the students had to verbalise their thoughts, they had to organise them in a logical sequence to justify their thinking and mathematical reasoning. If the students were experiencing difficulties Sylvia either assisted them immediately or she left them with a challenging question allowing them time to think, returning after a few minutes to continue the conference. Sylvia also used questions to provide a structure for supporting students to work mathematically and independently.

Most of the mathematics investigative tasks were set for individuals to work within a group environment to make meaning through interactions with their peers and with the teacher. The students were able to choose the point where they began the task and what strategies they used,

depending on their prior knowledge and the material they had at their disposal. Students worked collaboratively or co-independently, however co-independent work was more prevalent.

Sylvia used Growth Points as a conceptual framework for assessing student's progress, planning a unit of work, designing multi-entry tasks, thinking of possible directions students could take as they investigated, and possible questions she could ask to determine what they understood as well as to challenge them. During a discussion at the end of the study Sylvia commented that Growth Points gave her a better understanding of measurement (linear measurement and area) and individual students' progress, giving her a deeper understanding of individual students' needs and growth.

In Sylvia's room the physical setting, the social setting and the mathematical experience worked together to support all students as they developed and built mathematical understandings and mathematical ways of thinking.

Students' Responses

To address whether the teaching strategies were effective for all students as they constructed their understandings in mathematics, this section will focus on students with special needs, LBOTE students, and high achieving students. Some of these students' performances in the classroom will be discussed along with their test results. This section will briefly outline the responses targeted students made to the mathematical experience, through the use of work samples and tests results.

Special needs students

Ruby and Craig were 2 special needs students in Sylvia's class and Robert was in Zoë's class. At the beginning of the research the teachers were asked to describe their students as learners. This is Sylvia's description of Ruby and Craig.

Ruby was diagnosed with phenylketonuria (PKU). This condition can affect her learning by demonstrating mild to moderate impairments in a number of areas, including auditory/ verbal attention span, visual sequencing, and verbal acuity, learning and attention control – the ability to switch attention.

Craig has a mild diplegia which is a result of cerebral palsy. His mobility issues are quite mild, but he has difficulty with his fine motor skills. This condition can affect his learning as he tires very quickly. His fine motor skills are poor; hence he has difficulty with recording. Craig received about ten hours per week support time from the School Support Officer.

Ruby is from a low socio-economic, single parent family and receives school card benefits. Ruby experienced difficulties in most subject areas, but especially in mathematics. At the beginning of the year Ruby was a learner who required prompting from the teacher in order to complete a task. She waited for Sylvia's input before starting an investigation. While she often displayed some sound mathematical ideas and skills she still needed to be led through the task.

Craig had a high absenteeism rate, being away for 60 days during the period of the research. Sylvia described Craig as a student who was keen to complete most tasks but usually required the teacher's assistance throughout the process. He had difficulties manipulating some of the mathematical equipment.

The test results shown in Table 9 and Ruby's work samples (Figures 36 to 40) indicate that these students made significant progress over the 9 months of the research.

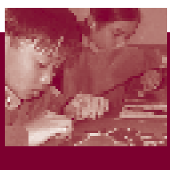




Table 9. Ruby and Craig: Pre-test and post-test percentage scores for linear measurement and area

Name	Investigation	Linear measurement		Area	
		March	November	March	November
Ruby	Practical observation test	28	100	25	65
	Pencil-and-paper test	55	80	29	45
Craig	Pencil-and-paper test	27	60	14	45

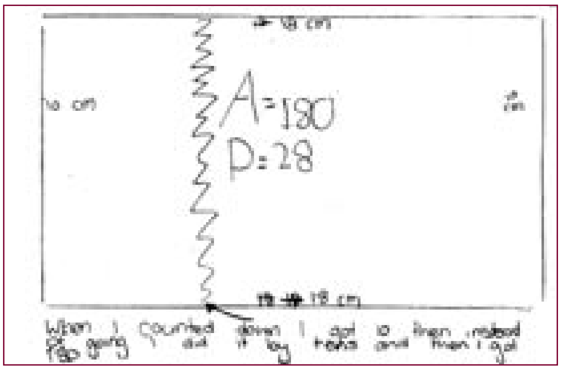
Note: As Craig was unable to complete the practical observation pre-test, his results were not included in the post-test.

Ruby

The pre-test results demonstrate that Ruby had difficulty with parts of units in both area and linear measures; confusion between area and perimeter attributes; and difficulty with number knowledge linking with measurement. Sylvia worked with Ruby over the nine months supporting her to use her existing informal knowledge and to link it to conventional mathematical knowledge. While affirming this informal knowledge Sylvia's facilitation challenged her to refine and build onto it.

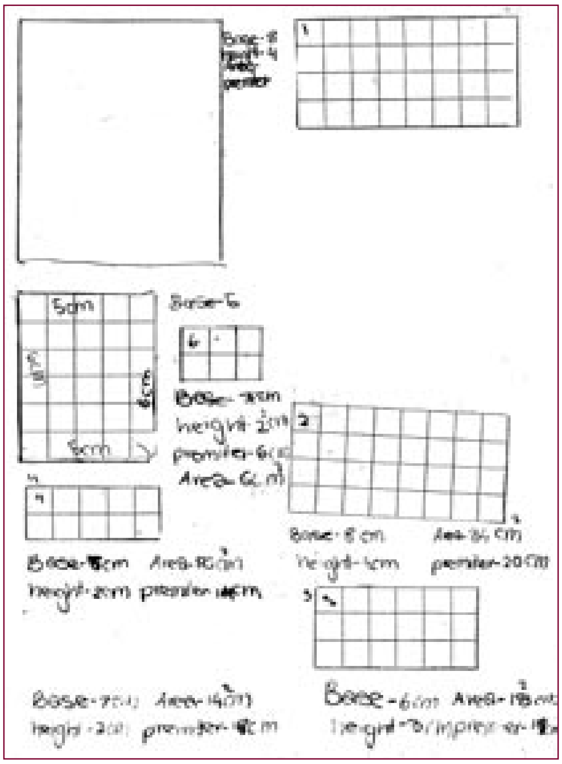
The following example illustrates how Sylvia supported Ruby to build onto her existing knowledge of area. At the beginning of the unit Ruby was tracing around shapes using grid paper and then counting units individually to work out the area. Later in the unit it seems that Ruby began to realise that the units could be counted in groups, but she had not yet recognised that the linear measures could be used to calculate area. The work sample in Figure 36 shows that Ruby traced the rectangle, then covered her shape with a transparent centimetre square grid and began to observe groups of 10 within the rectangle. She counted in groups of 10, that is, 10, 20, 30, … 180, rather than noticing that there were 18 groups of 10. Ruby then traced the rectangle in her book, recorded her measures of linear lengths and the area measure, and wrote how she had counted in groups of10 to determine her area measure.

Figure 36. Ruby: Counting by tens to find the area



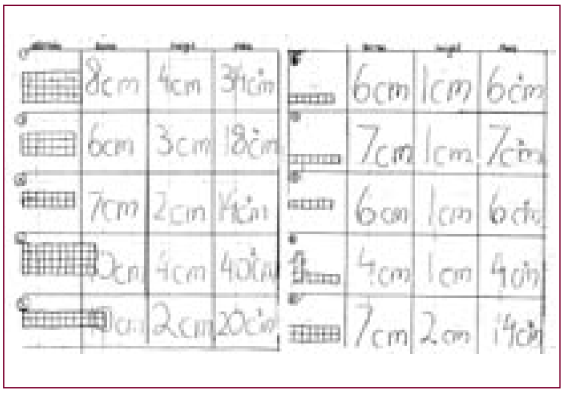
Sylvia's analysis of Ruby's work sample was followed by a discussion. Sylvia decided to set a new challenge to encourage Ruby to refine and build onto the use of group counting in an attempt to find the rows and columns, thereby leading her to the area formula. She asked Ruby to construct 20 different rectangles and to collect data on the linear dimensions and the area. Sylvia's question was: *Can you find a pattern in your data that will support you to find the area more efficiently?* The work sample in Figure 37 illustrates some of the data Ruby collected.

Figure 37. Part of Ruby's data for the area of rectangles



Ruby's data extended over several pages as she found it difficult to see a pattern. Sylvia encouraged her to put her data in a table (see Figure 38), but she still failed to see the pattern.

Figure 38. Ruby's recording of her area data



Excerpt 44 is the dialogue between Ruby and Sylvia after Ruby had reorganised her data into the table and was trying to search for a pattern.

Excerpt 44

Ruby	*I think I have found a pattern but not in all of them. I found it in 1 cm by 6 and 2 cm by 7. You see 1 times 6 is 6 and seven 2s are 14.*
Sylvia	*What does the times mean?*
Ruby	*Groups of? 7 groups of 2 is like plussing 7 + 7 … it's 14.*
Sylvia	*Can you see any groups in your diagram?*
Ruby	*Yes, 2 groups of 7 [Ruby points to the base line and moves her fingers up and down the groups.]*
Sylvia	*Now look back at your table data. Can you see any other examples where this might be happening?*
Ruby	*No.*
Sylvia	*What about this one? [Pointing to the data for a rectangle that has height 4 base 1 and area 4.]*
Ruby	*No.*
Sylvia	*How are 4 and 1 different to the 1 and 6, 6 you mentioned before?*
Ruby	*It's less than 6 and 1.*
Sylvia	*Yes, but what was the pattern you saw with the 1 and 6 and 6?*
Ruby	*Oh, 6 groups of 1 is 6.*
Sylvia	*So what would it be for this one? Does that same pattern happen here?*
Ruby	*Oh, yes, it's 4 times 1.*

Realising that Ruby's number skills were not supporting her to see the pattern Sylvia encouraged Ruby firstly to reflect on her knowledge of 'groups of' and 'times', then she refocused her on the visual patterns in the diagrams in her table of data as she attempted to support her to link her knowledge of 'groups of' to the pattern in her table.

Sylvia:	*Can you look back through your data that are not by 1 and see if your pattern still holds?*
Ruby	*[pointing to one of her diagrams] It's 10 and 4 … 40.*
Sylvia	*Does your pattern work here?*
Ruby	*I think so. I think there might be 10 groups of 4.*
Sylvia	*Can you cut out this rectangle out of grid paper and then cut out the groups.*

Sylvia left Ruby to investigate further, returning to her later when Ruby had cut a 10 by 4 rectangle into 10 columns of 4.

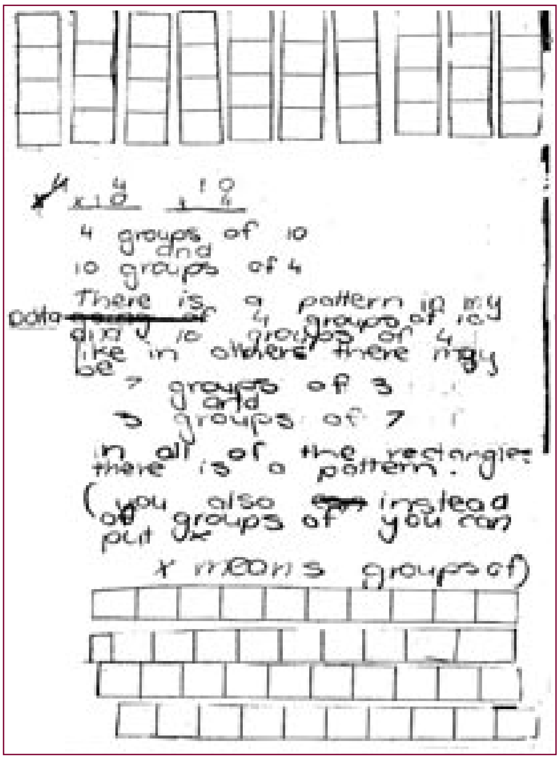
Sylvia	*What have you done now?*
Ruby	*Cut out the 10 groups of 4.*
Sylvia	*Can you see any other groups?*
Ruby	*Yes, 4 groups of 10.*
Sylvia	*Where are they?*
Ruby	*Here [running her finger along the rows that she had already cut out].*
Sylvia	*What measure of your rectangles tells you about the groups? Look back through your data.*
Ruby	*The height and the base tell you the groups, that' s why you can times them.*
Sylvia	*Is that so for all your data?*
Ruby	*Yes.*

Ruby started to make the connections, but to ensure that she understood the pattern and the relationship between the linear measurement of a rectangle and its area, Sylvia gave her another task.

Sylvia	*Can you draw up some rectangles on blank paper and work out the area? Before you do, could you think carefully through what you have just discovered and record all that you just found out so you can reflect on this information later?*

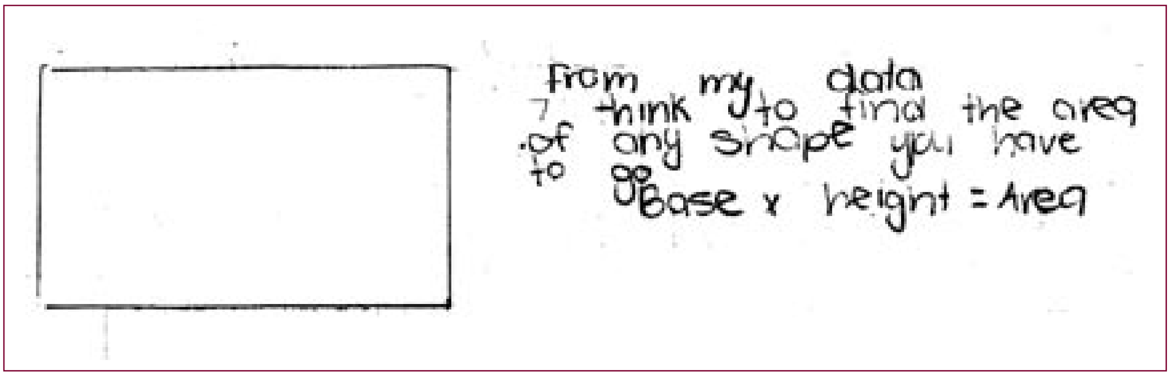
Here Ruby has described the pattern she could now see occurring in the different rectangles. She used her descriptions for one rectangle to make a generalisation for other rectangles. At the end of the session, Ruby recorded her reflections and new understandings (see Figure 39).

Figure 39. Ruby's recording of her understanding of how to calculate the area of a rectangle



Once Sylvia saw that Ruby recognised the connection, she focused Ruby's attention on the relationship between the linear measures and the grid of columns and rows that formed the groups. Figure 40 shows Ruby's work samples from the sessions indicating how she used her own informal ideas and language to build the understanding of the conceptual ideas behind the formula $b \times h$.

Figure 40. Ruby's recording of her conceptualisation of the rectangle area formula



Here Ruby has linked her idea of columns and rows to the linear measures of the rectangles. She has worked out the area of rectangles using blank paper and her ruler. In this sample Ruby is now using appropriate units for both linear measurements and area. She has generalised her new insight of base times height as applying to any shape. However, after further investigations, she came to understand that $b \times h$ would apply to other rectangles. In the post-test Ruby's overall results indicated significant growth. Ruby selected the formula ($b \times h$) to solve several questions relating to rectangle area, but she did not choose the formula when calculating areas of other shapes.

In the practical test, students had to investigate the area of several cards to find out which of the 4 cards would need more paper to make. Ruby selected the formula to calculate the rectangle shaped cards. For the other cards, including the triangle shaped card, she used the centimetre-squared grid and counted the squares. Ruby was observed counting out the whole units, and then worked out the parts of units by piecing them together to make whole units. She was able to use standard measures for both linear measurements and area. She was able to convert all of the linear measures and determine how many square centimetres in a square metre. Ruby also began to use decimal fractions to represent parts of units.

Craig

The pre-test data (see Table 9) shows that at the beginning of the research Craig's knowledge of area and linear measurement was extremely limited. While he was able to answer some of the linear measurement questions correctly, he had difficulties when estimating lengths. He was unable to use a measuring scale unless the object being measured was lined up at zero. Craig's test results indicate that he was unclear of the term *perimeter* and was unable to convert from one unit to another. Craig was also unable to solve any test questions relating to area. In the practical test Craig appeared not understand the task and made little attempt to complete it. It could be, however, that his performance was affected by the presence of an unknown person administering the test.

As was the case with Ruby, Sylvia ensured that Craig was supported to work at his own pace, encouraging him to bring his existing knowledge to the classroom work and then tired to support him to build onto it. The difficulties Craig experienced were often in manipulating the mathematical tools. On many occasions Sylvia needed to rethink the task to ensure that he could physically manage it. Craig often worked with an assistant teacher who supported him with some of the physical aspects of the tasks. For example, when he wanted to measure the perimeter of an object

with string, the assistant teacher held the string as Craig measured. The assistant teacher was often the support person, although Sylvia and other students supported him at times.

Craig's recordings were difficult to read and computer access was limited. As a result we did not obtain any meaningful recordings to illustrate his thinking and understandings. In the post-test results 9 months later, however, Craig had made significant progress, as shown in Table 9, given that at the start of the year his understanding was minimal.

Robert

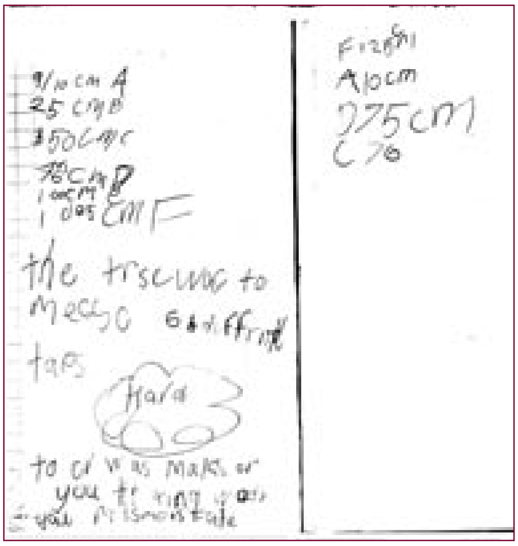
Robert was identified as special needs and received funding for educational support for his emotional needs. Zoë described him as having limited writing skills, which was also backed by a report stating he needed support time to focus on handwriting and letter formation, size and spacing.

Robert always selected a seat that was very difficult for the teacher to access and he did not speak out in class discussions. Zoë worked with Robert to encourage him to build onto his verbal skills and develop his written communication. He was also diagnosed as needing open-ended timeframes in order to diminish his feeling pressured.

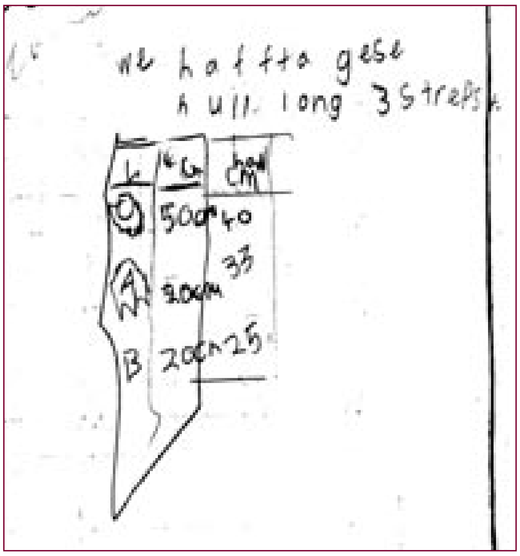
Figure 41a shows how Robert recorded his findings and his thinking in his own words. Once the class had discussed tabulating information as a strategy to see patterns in data and to show information effectively, Robert went on to tabulate the data he collected in the next piece of work but had difficulties with drawing tables (see Figure 41b).

Figure 41. Robert's recordings of his data

a



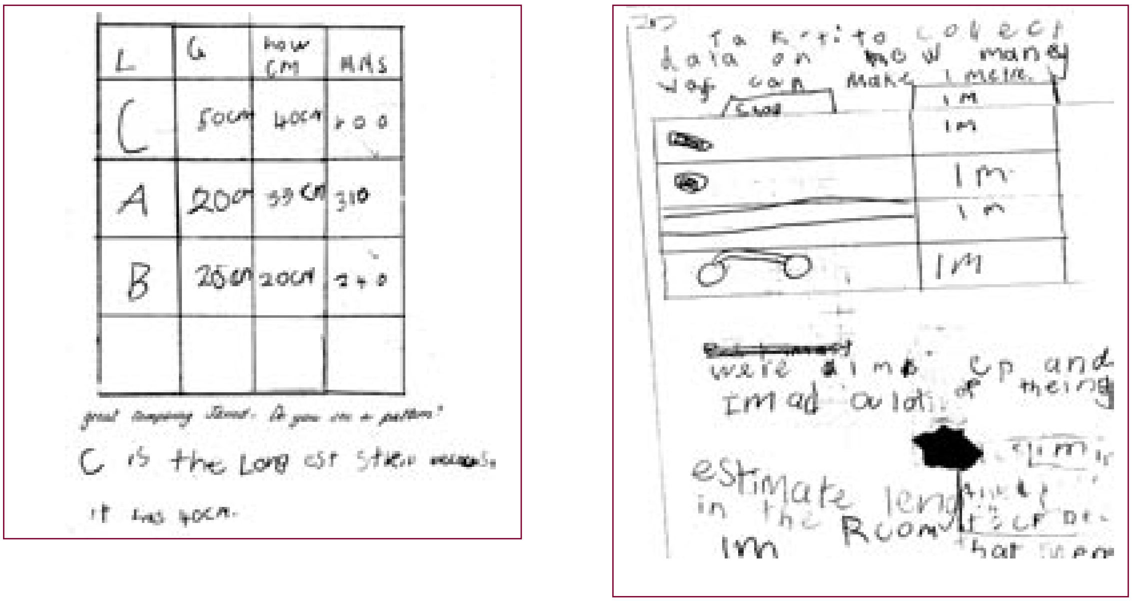
b



The teacher assistant, seeing his attempt, drew a table for him where he recorded his data (Figure 42a). In the following lesson, Robert made his own table and recorded his findings without assistance (see Figure 42b).

Figure 42. Robert's record of his findings

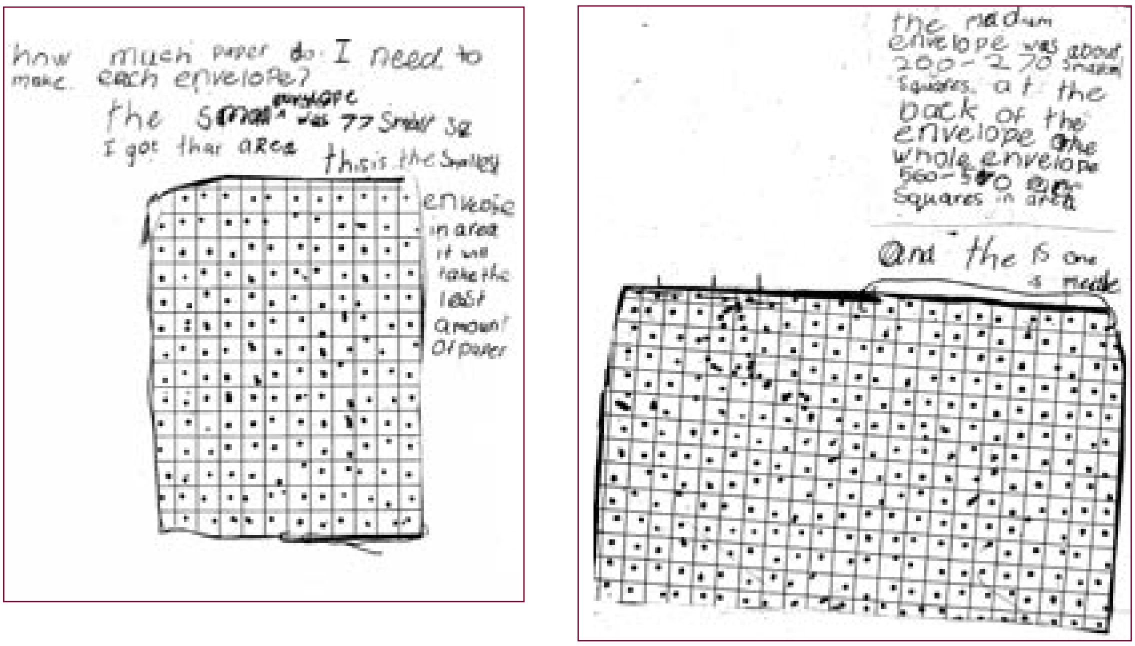
a



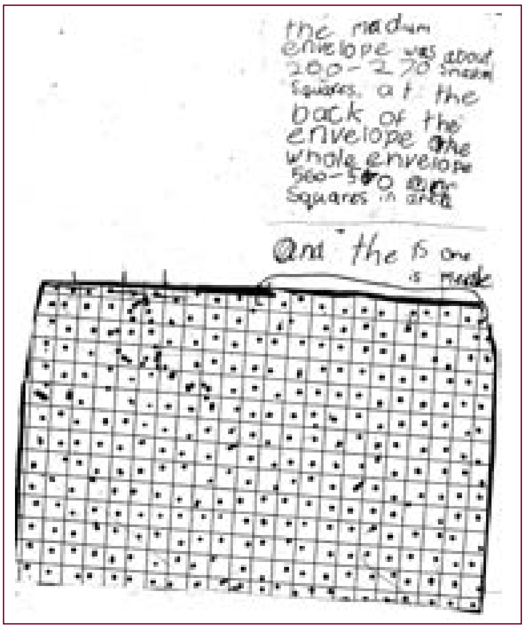
The next set of work samples (Figure 43) shows how he worked out the areas of different-sized envelopes and how he used his own language to describe his findings.

Figure 43. Robert: Finding the areas of envelopes

b



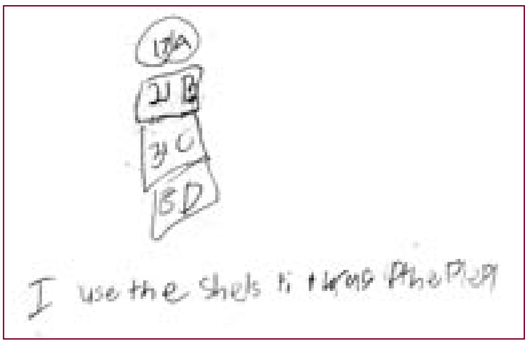
c



Robert's pre-test response (Figure 44a) shows that he understood the task was to measure the space occupied by each card. However, he used non-uniform units (shells) to measure area, hence showing a lack of understanding of the need for uniform units, that the units had to be of the same size and there should not be gaps and overlaps. In the post-test (Figure 44b) Robert used standard units to measure and was able to communicate his findings more effectively. He was also able to relate his findings to the question.

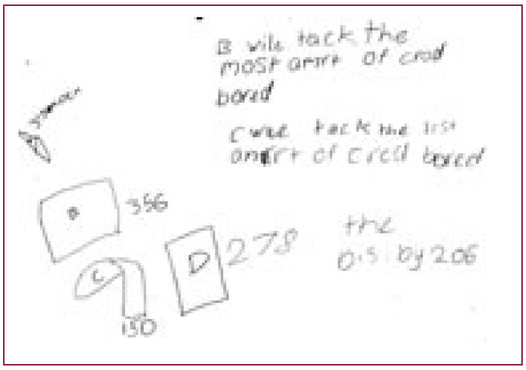
Figure 44. Robert: Area investigation test

a



Pre-test (March)

b



Post-test (November)

Figure 45 shows Robert's recording in the post-test linear measurement investigation.

Figure 45. Robert: Linear measurement post-test

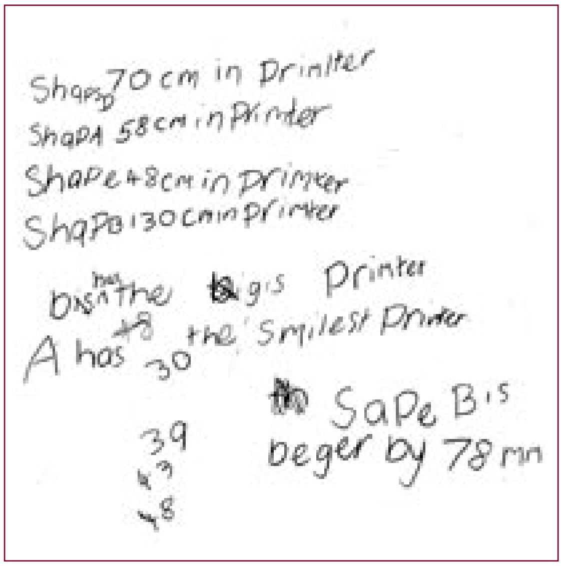


Table 10 shows Robert's pre-test and post-test scores for the pencil-and-paper test and for the investigative test. Overall Robert's performance in both the written and investigation tests improved. His investigation pre-test score reflects his failure to understand the task.

Table 10. Robert: Pre-test and post-test scores in the pencil-and-paper test and the investigative test

	Linear measurement		Area	
	March	November	March	November
Pencil-and-paper test	36	69	20	70
Investigation	28	78	0	55

As the year progressed, Robert was more willing to participate in small group as well as whole class discussions. Zoë changed the seating arrangement in her class so that he was easily accessible and he started to interact more with her. Overall, these examples and the test results show that the performance and participation of the students with special needs improved.

Language Background Other than English students

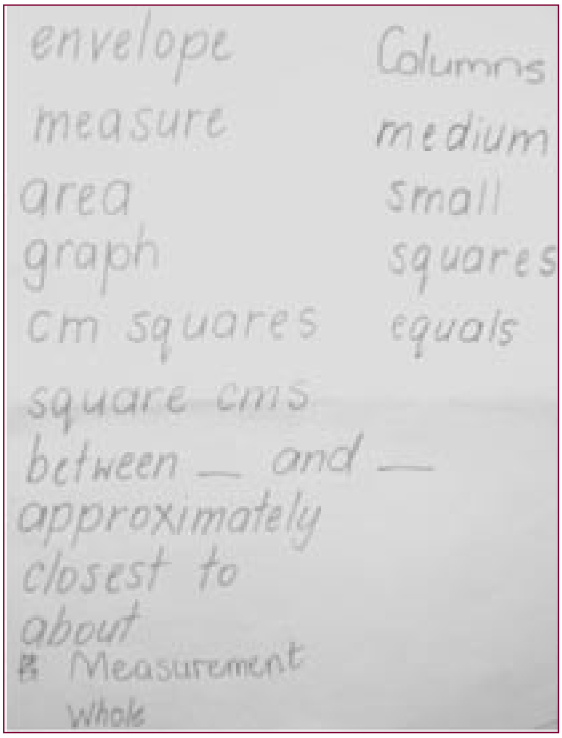
Of the 3 classes whose data we have analysed, Tania's had the most LBOTE students, though there were some in Zoë's Year 3 class. Just over a third of Tania's class spoke English as a second language. The pre-and post-tests data show that the LBOTE students' performances were significantly higher than other students at all levels, though the most significant growth happened in Year 4. These students' performances were not only significant in the pencil-and-paper test, which included multiple choice questions and short answer questions, but also in the practical test where their performances were judged on a Rasch partial credit system where the skills tested included communication and problem solving.

The strategies identified in the data in both of these classes that we believe supported the LBOTE students' growth included language support, the valuing of students' experiences and informal language and the one-to-one interaction with the teacher and peers. To support LBOTE students both teachers ensured that in each group there was at least one native English speaker who could read and understand the instructions and could be used as models. However, the observations indicate that many of the LBOTE students were supportive of the English-speaking students in terms of mathematics as well as English language.

Both Zoë and Tania spent time at the beginning of new units of work familiarising the students with the language and terminology associated with the unit. Zoë, however, did so at the beginning of every lesson. This often took the form of a class brainstorming or discussion around words the students thought they might encounter or want to use. These brainstorming sessions provided opportunities for the students to link their own language with the conventional language of mathematics and to unpack the meaning behind the terminology. Zoë created word lists (see Figure 46) as the students brainstormed. These were displayed for reference throughout the unit of work and were added to as new words came up through investigations. There was at least one native English speaker was in each group, to support LBOTE students. The discussions seemed to support the students in thinking about contexts outside of the mathematics classroom where that language or mathematical idea may occur and whether it had a similar or different meaning. They were then able to see the connections with their everyday life. Tania invited parents to talk to the students of the ways they used mathematics and the mathematical words they used in the home and at work.



Figure 46. Example of a word list displayed in Tania's classroom



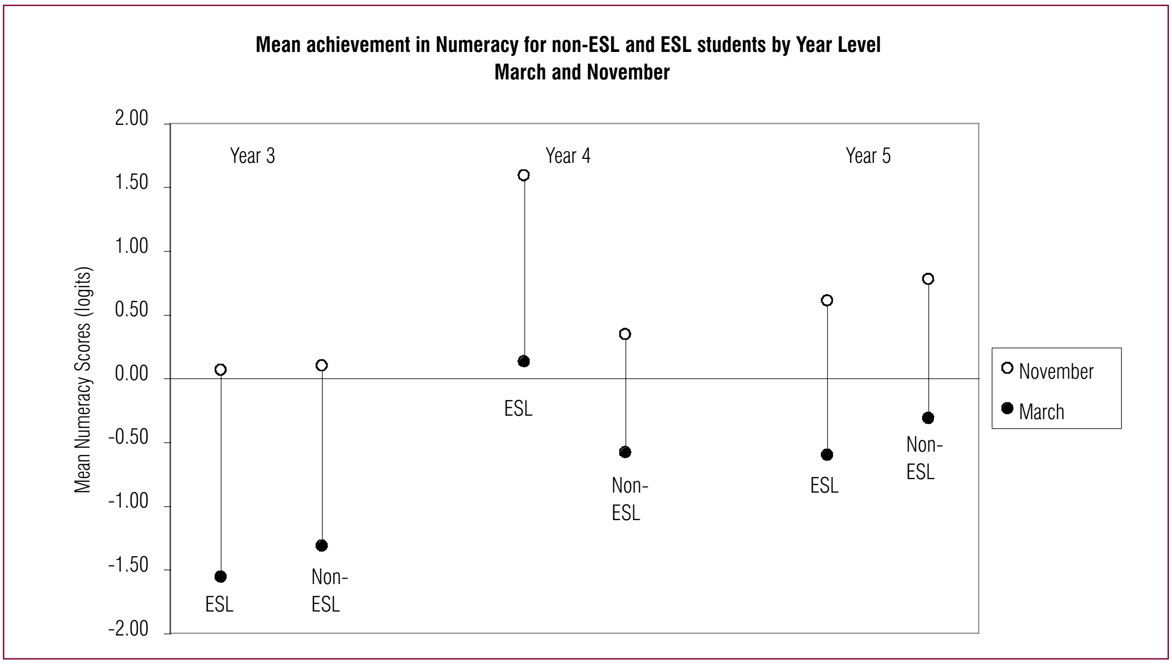
Test performance of LBOTE students

As shown in Figure 47, the mean Rasch-scaled scores of LBOTE students and non-LBOTE students increased from March and November for all 3 year levels.

Four of the Year 3 LBOTE students were in Zoë's class. As the performances of LBOTE students were analysed by year level rather than by teacher, we cannot determine the performance of the LBOTE students in Zoë's class. However, as indicated previously, all students in Zoë's class, including the LBOTE students and students with special needs, made significant improvements.

Four of the Year 4 LBOTE students and 6 of the Year 5 LBOTE students were in Tania's class. All students in Tania's class improved significantly, implying that the strategies she had in place supported the LBOTE students as well as the other students. It must be noted, however, that although LBOTE students formed just over a third of Tania's class, the overall

Figure 47. Mean achievement in numeracy for non-LBOTE and LBOTE students by year level, March and November



number was small. Care must be taken when comparing their performance with the students whose first language is English.

Tables 11, 12 and 13 show the test scores of 3 LBOTE students. Pamela was one of Zoë's case study students, while Nhi and Trang were Tania's case study students.

Table 11. Pamela's growth in achievement in linear measurement and area from pre-test to post-test

	Linear measurement		Area	
	March	November	March	November
Pencil-and-paper test	29	85	20	80
Practical observation test	0	83	0	63

Table 12. Nhi's growth in achievement in linear measurement and area from pre-test to post-test

	Linear measurement		Area	
	March	November	March	November
Pencil-and-paper test	85	93	33	67
Practical observation test	22	100	42	85

Table 13. Trang's growth in achievement linear measurement and area from pre-test to post-test

	Linear measurement		Area	
	March	November	March	November
Pencil-and-paper test	64	92	78	89
Practical observation test	56	100	75	95

As all students in each of the 3 classes improved, we can conclude that the teaching strategies that the teachers had in place supported all of the students, including those with special needs, the LBOTE students, and the more able students. It is likely no single aspect of the teaching strategies or the environment was responsible for the learning for all students. Rather it was a combination of a number of factors, including the classroom environment, the conferences, the students working together, the open-ended nature of the tasks, the teachers' reflections on students' work, and their own pedagogy.



Combined Case Analysis: Emerging Issues and Understandings

Despite the unique nature of each case study and teacher, there are shared strategies that are evident and this chapter is an attempt to identify what those emerging commonalities are.

Reflecting on the complexity of the mathematics classroom

The environment the teachers created in their rooms was one of respect, cooperation and challenge. These strategies employed by the teachers created an environment where students were supported to participate actively, be challenged, and challenge themselves and others.

All three teachers worked from the premise that students learn best when they are encouraged to construct their own understandings. The organisation of their classrooms reflected the teachers' beliefs that learning is not solely an active construction of knowledge but is also influenced within a social environment.

As a consequence of this, students were organised into small problem-solving groups where they would share ideas and strategies, thereby entertaining the potentials in the use of several strategies to solve a problem and thus recognising the value of flexibility in thinking. Two of the 3 teachers organised their sessions into 2 or 3 distinct moments where students shared ideas, not only with the students in their small group, but also with the whole class if they wished.

The physical and social settings of the mathematics classroom

In all 3 classrooms the students' tables were organised in groups, while in 2 of the 3 classrooms there was an empty space at the front of the class for whole class discussion, brainstorming and sharing. In both of these rooms the students used this space when they wanted to work on the floor. Once the students were assigned a group they were then free to pair up with any student within their class. These organisations supported the students to share ideas and strategies and be active participants in the learning.

Grouping strategies

In all 3 classrooms the teacher decided on the composition of the group. While groups could be either mixed gender or single sex, all groups were mixed ability. This grouping strategy provided opportunities for all students, including LBOTE and special needs students, to enrich their mathematical experience. In both Tania's and Zoë's classes at least 1 student was placed in a group to support the LBOTE students with their English language. The mixed ability grouping provided opportunities for all students, including LBOTE and special needs students, to listen to others using the mathematical language. This therefore provided a model for students who were experiencing difficulties; it encouraged them to link their informal language and experiences to the conventional language.

The mixed ability grouping also exposed students to different mathematical ideas and strategies and different ways of thinking. This enabled the students to reflect on their own thinking as they connected their findings with those of other students. Within these small groups students grew in confidence to verbalise their thinking, to question others and to share their strategies.

Displays

Displays were used as a means of supporting students with conventional mathematics language as well as expose students to different alternatives and to illustrate that students' work was valued.



In each of the 3 classes selected words and phrases expressed during a brainstorm were listed and displayed. As the unit progressed, the teacher and students added new words to the list. This dynamic process, with students actively involved, scaffolded students' independence. Students' work samples were also displayed in all 3 classes. Displaying students' work samples exposed students to different working styles and mathematical strategies and different ways of recording mathematical ideas and thinking. It was also a way of valuing students' work and it provided a context for visual images of the conventional terminology through the teacher's labelling of the work.

The teachers changed the work sample displays as the unit progressed. In an attempt to ensure equity the teachers kept a record of whose work went on display so that all students had their work displayed at sometime during year.

Resources

All 3 teachers gave students access to a variety of materials they thought could assist the students to work at the concrete level and build on the concrete as they moved towards the abstract. Access to a variety of materials/ tools, for example standard and non-standard units, were deemed as important and were received in the classrooms as positive support for all levels of learning. Students were encouraged to pick any materials they wished from those selected and displayed by the teacher, or they were also able to select from the cupboards.

Classroom protocols

The social protocols operating in all the classrooms encouraged the students to interact with each other. During the mathematical experience students who needed assistance were required to ask a group member before seeking the teacher's support. This encouraged the students to work collaboratively and to respect each other's ideas and strategies. Students were observed giving each other explanations, asking each other questions and referring others to the list of words on display to ensure appropriate use of vocabulary and correct the spelling of words.

While all students had access to the teacher, there was a structure in place to ensure the one-to-one conferences between the teacher and another student were not interrupted. The protocols clearly explained a process for students to use when they needed help. These involved firstly, trying other methods themselves, secondly, consulting with other students and lastly, letting the teacher know they required help by placing an indicator where the teacher could see it.

In most classrooms the students kept to the classroom protocols. However, in Sylvia's class, one student sought the teacher's attention whenever he needed help, and in this instance she gave him the attention. During the reflective collaboration time Sylvia pointed out that this particular student had behavioural problems and if he was not attended to he did not do any work and would also stop others from doing so. She therefore ensured that he got the attention, but also made sure that he did not monopolise her time.

All students knew that they were expected to work on their investigations and to record their own thinking. Even when students discussed and shared ideas in groups they still recorded their own understandings in their own way in their own book. A vital, underlying assumption in all these rooms was that *all* students could learn mathematics.

The mathematical experience

Learning mathematics and becoming numerate entails more than learning mathematical facts. It is also being able to use mathematical knowledge flexibly to create understandings. Schoenfeld (1992) describes students' numeracy as having a 'mathematical point of view'. He asserts that one needs to acquire the knowledge and the tools to construct an understanding of the knowledge, which may then result in the construction of more knowledge.

The mathematical experience that the teacher creates for the students needs to take into account the tasks, how the task is facilitated and how the teacher supports the students during their investigations. These elements can assist the students not only in their knowledge construction but also in their understanding and ability to use the knowledge they have constructed more flexibly.

Mathematical experience: Facilitating the building of mathematical understanding

The tasks/investigations

All 3 case analyses reveal that teachers designed open-ended/investigative tasks for their classes. Open-ended/investigative tasks allowed the students to bring in their prior knowledge and understanding, so they could start their investigations from the point of their current thinking. That is, they could use their prior knowledge as starting points. The task also involved the students in the collection of data, searching for patterns, making conjectures and drawing conclusions.

When designing investigative tasks the teachers took into account each student's prior knowledge, which they established through observations and discussions, and/or by getting the student to record what they know on the topic. This was done in a manner that made sense to the student. Having this freedom, students were able to express themselves giving the teacher access to their understandings.

The teachers were also informed of each student's prior knowledge through pre- and post-tests scores and by analysing the results through the pedagogical lens of Growth Points. The 3 teachers also took into account each student's experiences and interests. Though they used their implicit knowledge of each student to determine the student's interest and experiences they also often engaged, with the collaborative group, in discussions about what the students were interested in, finding out from the students how they might conceive the task to make it accessible, and to keep the student engaged. As a result, students readily engaged with the set tasks. Once the task was set, the teacher moved around the classroom (referred to as 'roaming' in the case analyses) ensuring that the students understood the task and had a starting point.

As mentioned in the case analyses, students used different strategies to go about the tasks. Henningsen and Stein (1997) state that there are a lot of factors that can undermine classroom activities and one of these factors is the task itself. They assert that if there is a mismatch between students' prior knowledge, interest and motivation, then the students can fail to engage with the task. This could result in students having difficulty maintaining a high level of cognitive activity, as cognitive engagement is important. In an ethnographic study of a secondary mathematics classroom, Goodchild (2001) found that even though students may engage with the mathematical task, their engagement can be at a superficial level. Goodchild refers to this as 'blind activity', corresponding to Skemp's (1989) 'symbolic level involvement' (see chapter 1). Researchers such as Bennett, Desforges, Cockburn and Wilkinson (1984) and Henningsen and Stein (1997) go



further in saying that if the task is pitched at the students' realm of experience, the students stand a better chance of engaging at a high cognitive level.

Flexible environment

The cognitive demands and nature of the tasks undertaken in the 3 classrooms were such that students were engaged for meaningful periods of time. During these one-hour sessions all 3 teachers engaged the students in doing the task while moving strategically around the class conferencing selected students. As discussed in the case analyses, the tasks did not end at the end of a mathematics lesson; they often went well beyond the scheduled time, and when students put away their investigations at the end of the session it was to continue the work in the next lesson. Often at the end of a 1-hour session, as was the case in most of the 3 classes, students and teachers were so involved that the students complained when they had to stop.

As the students had a range of resources to choose from the teachers were in a position to observe their choice. This gave them insights into what the student knew about units. The teacher could also elicit their thinking and problem-solving skills which could be used as indicators of prior knowledge. The range of materials provided included both standard and non-standard units. This meant that the students were able to start on the activity at a point where they felt comfortable. They had to make decisions and be flexible in their choice, hence encouraging them to take risks.

For example, when 1 group of students was asked to compare the area of 3 different objects, a few students employed 3 different units. The question for the teacher was, 'Do the students understand that to make a fair comparison they have to use the same unit, or do they think they have to use more than one unit as they were given more than one object?'. If the students in this instance were only given 1 unit to use, then the teacher would gain no clear information regarding the students' understanding of units. Also, having to make decisions about which unit to use when measuring indicates to the students that there are several possibilities when measuring, and that one uses the most efficient and appropriate unit for the task. In this way students may be able to transfer this knowledge to other situations.

Because the students were exposed to a range of materials to use in their investigations, the teachers were able to gain insights into the students' understandings and thinking. Hoyles (1990, pp. 124–125), in her discussion of classroom resources, noted that 'different resources lead to different representations of concepts: they evoke different structures and support different reasoning patterns.'

As pointed out by Doyle (1986), the resources students use to make sense of the task can impact on what the student learn from the experience. For example, in 1 class the students collected data on rectangles and were invited to find out whether there was any relationship between the dimensions. Some of the students, using 'flip blocks,' saw the rows and the columns but had difficulty translating the two dimensional aspect of the 'flip blocks' to the linear measurements. In other cases, where students used centicubes, such relationships were easier to understand. Tania, for example questioned a student about his use of a $2 \times 2 \times 2$ cube to work out the area of a fishpond. The student was able to justify the use of this manipulative, explaining that because each face of the cube is 4 cm^2, working out the area is like counting in groups of 4. In these classrooms, the teachers worked with the students to help support them to understand their findings and to ensure that all students were constructing the same meaning and thus coming to a shared mathematical understanding.

Interactions

Zoë organised every lesson into 3 distinct moments – introduction, student investigations and sharing times. In the introduction and sharing time students engaged in discussions about their ideas, strategies and use of language. In all whole class interactions the teacher led the discussions or asked questions to facilitate the student's explanations of their work. While the teacher conferenced a student, other students interacted with each other during their investigations, supporting each other to solve the problem.

In Tania's and Sylvia's classes the 3 distinct moments were not as defined and they were not always present. However, in all 3 classes the students worked both together and individually within group settings. They worked either co-independently or collaboratively. During student investigation times, the students were conferenced by the teacher either individually or in small groups.

Working styles

Students were always seated in groups and they were encouraged to support each other during their investigations and problem-solving. Students discussed and shared mathematical ideas, strategies, and patterns/findings as they collected data or as they solved their problems. In Sylvia's class, however, some students tended to pair up informally, whereas others worked on their own and used the group as support. Within these mixed ability group settings students tended to support each other (peer support).

Students also had the opportunity of working co-independently or collaboratively at any time during the lesson. Using the co-independent style, the students decided when to use the resources available to them, namely their peers, their teacher and/or the available mathematics materials. The students were able to use their own strategies as they constructed and recorded their own understandings. This co-independent style gave the teacher insights into individual students' thinking. Being in small group settings regulated by the classroom norms, all the students worked together, but they had the flexibility to choose their materials and to work the way that best suited them for the task. In Sylvia's class, for example, most students tended to work in pairs though they each wrote their findings in their own way in their own workbooks, hence giving the teacher access to individual students' understandings.

Conferencing

All 3 teachers spent considerable time interacting with students on a one-to-one basis, which is referred to in this report as *conferencing.* A conference is a conversation between teacher and students in a non-threatening environment, creating an atmosphere of mutual trust where the teacher respects the students' ideas and encourages the students to bring in their prior and informal knowledge. Being sensitive to the students prior knowledge and 'natural inventiveness' during a conference, the teachers' questions engaged the students with the mathematical ideas and supported them to link their informal and prior knowledge to other ideas as they constructed mathematical understanding. As a result, students were not inhibited to bring in their own experiences. The interactions during a conference provided the students with a forum to talk individually about their mathematical understandings to the teacher and for the teacher to affirm and challenge their thinking and strategies. Conferencing provides a context in which the student and the teacher can establish common understandings. What develops between the teacher and the students during a conference can be conceived of as a collective and shared understanding through the development of a common language and a discursive context (Edwards & Mercer, 1987).



In Zoë's and Tania's classes there were instances where the students had difficulties linking their present knowledge to their prior experiences. Both teachers were observed to ask closed questions (described as 'funnelling' – see Bauersfeld, 1980). The questions were direct and appealed to the students' past shared experiences. During a conference in a sharing time (whole class setting) Zoë often summarised what the student said into succinct phrases, which she then repeated to the student. This strategy ensured that the student identified what was important as well as developing a shared understanding with the teacher, and later with the rest of the class. The concept the student formed, however, had to be compatible with those of other students in the class and with the mathematical community. The conferences therefore created an opportunity where students could develop a shared, or common understanding (Edwards & Mercer, 1987) of mathematics concepts with the teacher and subsequently the class. When the conference was in small groups, then the group would share the teacher's and other group members' understandings. In Zoë's class, where conferencing also occurred in the Sharing Time (public conference), whole class sharing reinforced the shared knowledge.

Reflecting and verbalising thinking

Before starting a conference all 3 teachers determined the range of the students' existing knowledge and understanding. Zoë observed the students at work before she started a conversation, while Tania and Sylvia questioned the students directly to encourage the students to verbalise their thinking. The students' responses to the teacher's initial questions gave the teacher access to their thinking and understanding, as well as to the strategies they were using. Having established the students' thinking the teacher then formulated a series of questions to support the students to link their prior knowledge to the concept being investigated, or to bring in this prior knowledge to help solve a problem. The teacher's questions were different in each context, as the students' strategies and resources depended on their prior knowledge and understanding. The type of questions the teacher asked influenced the cognitive demands placed on the students as they built their mathematical understandings.

The teachers' questions therefore enabled the students to think about their thinking (Skemp, 1989). Towards the end of the year, the students in Sylvia's and Tania's classes were observed asking similar types of questions of their peers and of themselves. By being encouraged to reflect on their prior knowledge the students learnt to recognise how this knowledge could be used in a particular situation.

Encouraging students to reflect on thinking

Opportunities for students to learn also arose from having to communicate their thinking to the teacher and to their peers. During a conference the students had to express their thinking, either orally or in written form in their workbook. Having to verbalise their thoughts forced the students to organise their thinking in a logical sequence so that it made sense to them, the teacher and their peers. When the students' communications were unclear the teacher probed further encouraging the students to expand on their explanation. Wood and Turner-Vorbeck (2001) found that in a socio-constructivist classroom context the expectation is that students not only tell how they solved the problem, but that they were expected to give reasons and clarification to their thinking.

In all three classes in our research project, the expectation was also for the students to express themselves and to justify their thinking, so that their thinking could be clarified and organised logically. This was a difficult process at the

beginning of the year for some students in all classes because students had difficulty articulating their thinking. By using questions, however, the teachers supported the students to recognise and reflect on their prior knowledge in relation to the new knowledge/problems. The teachers also challenged the students' thinking and strategies and supported them to move into their zone of proximal development (Vygotsky, 1978).

Asking challenging questions to support learning

The challenging questions the teachers asked required the students to explain and justify their thinking and the strategies they were using. Having to justify their thinking forced the student to reflect on that thinking and to provide a clear explanation. In certain cases the student responses indicated 'misconceptions', or inefficient strategies, and the teacher's questions put the student's responses in conflict with the student's current thinking. The conflict then forced the student to rethink. In some cases a student was given another task, which was used as a means of creating conflict. Rogoff (1994) describes how reflection can emerge when a conflict or disagreement occurs. In this case the teacher created the conflicts, forcing the students to critically examine and justify their explanations.

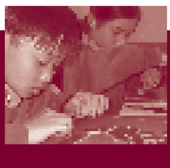
Listening to the students' explanations gave the teacher an understanding of individual students' needs and enabled the formulation of specific questions to meet those needs. The teacher therefore asked different questions of different students. However, at the end of the unit of work all the students shared more or less the same understanding. Throughout the conferences the teacher focused on the individual students' understanding, needs and growth. Though all the students in the class did the same investigation they did not always go about it in the same way, the strategies they used and how they went about it depended on their prior knowledge, the strategies they wanted to use, etc.

The cognitive demands in these classrooms not only encouraged students to think through a problem for themselves, but also challenged them to make decisions about solving the problem. When investigating mathematical concepts students had to learn to organise their data, to look for patterns, make conjectures and draw conclusions. In cases, for example, where the students knew that the length of a rectangle multiplied by its width gives the area, they had to show why the formula worked. The result of these challenges engaged the students in reflective thinking as they constructed their own understanding.

Whole-class sharing strategies

All 3 teachers provided a forum for the students to share strategies as they felt that the students needed to be aware that different strategies could be used to solve any single problem. At the initial stage of the research when Zoë and Sylvia introduced a new investigation they each discussed possible strategies with the whole class. As the year went by Sylvia realised that the students were working at different speeds and were going in different directions, so she rarely got the students together to discuss possible strategies at the beginning of a lesson. Zoë, on the other hand, continued to hold whole-class sharing strategy sessions throughout the research. Sylvia discussed possible strategies with students in small groups, or individually during a conference. Tania, on the other hand, did not engage the students in whole-class discussion, although the students shared strategies within their own groups and in some instances students moved to other groups.

Zoë's sessions, as illustrated in the case analyses, were in 3 phases. All of the sharing time and some of the introductory sessions were devoted to students sharing their thinking to the whole class. As the students explained their thinking, Zoë explicitly reiterated their responses. This was an attempt to ensure that the students developed a


shared understanding and that important points during the explanation were identified. What often resulted, however, was that the next day all of the students started to use those strategies, for example, the use of tables when organising data. In this way Zoë provided a range of models rather then modelling just one. Often the discussions that followed explored the relationships between the different strategies and their effectiveness and efficiency.

In all 3 classes the students shared strategies with their peers as they worked collaboratively or co-independently with other members of their groups.

Encouraging students to record using their own language

All 3 teachers consistently encouraged the students to make decisions about how to enter the task and how to record their understandings in ways that made sense to them. The emphasis was not just on recording their answers, but the students had to explain their thinking through their recordings. As a result the teacher was better able to access to the students' thinking placing her in a better position to support the students. Having to record their thinking in their own language the students had to organise and clarify their thinking, an aspect that the students had difficulties with initially.

A list of words inclusive of students' own informal language and the conventional terminology – resulting from brainstorming and class discussions – was prominently displayed. It provided support for the students to link their own language with that on the displayed list and encouraged them to use the new language. Discussions, links between the language and terminologies, and the class list, encouraged the students to incorporate conventional mathematical terms into their own language. Encouraging the children to write their thinking in their own language meant that they were not inhibited and their recordings therefore enabled them to formulate and clarify their thinking.

Using the students' informal and prior knowledge as a starting point, the teacher supported the students to make connections to the new concept and to conventional mathematical language. For example, Zoë got her class to make a square metre using newspaper. At the end of the session, she told the students that they could use it to measure the floor space of their bedroom. The next day Zoë asked the students to work out the area of the classroom, but they were to estimate its area before they measured. Student groups discussed possible estimates, but when they suggested 7 square metres, Chloe quickly said that it could not be so as the classroom was bigger than her bedroom and her bedroom measured 9 and a bit square metres. Being certain of the area of her room Chloe was able to appreciate the connections. As their understandings developed so did their abilities to transfer and to make sense of the mathematics. Baroody (2000) emphasised that to promote meaningful learning students must be assisted to relate their school-taught procedures to their informal everyday experiences.

Teachers' use of questions to scaffold thinking

During the conferences all 3 teachers used questions to encourage students to explain their thinking and understandings so that the teachers could access, build on, refine and/or challenge the students' thinking. The types of questions the teachers asked are detailed in all 3 case analyses. These questions were often similar, although some of the intentions behind the questions were different.

At the initial stage of a conference all 3 teachers asked the students reconnaissance questions, which required the

students to reflect on and explain their current reasoning. Once the teacher had accessed the students' thinking, subsequent questions supported students to link their prior knowledge to the concept being investigated as they built onto their current reasoning. These later questions were characterised by 'Why?' and they appeared to promote reflective thinking and to support the students to link their strategies/understandings to more sophisticated and conventional ones. Questions also supported the students to refine and build onto their mathematical thinking and strategies.

Supporting students to think and work mathematically

All 3 teachers engaged the students in mathematical enquiries where they had to organise their data, search for patterns, make conjectures and test their conjectures. This often resulted in the students writing their own generalisations, which, in some cases, led to a formula. The questions were directed to such an outcome, encouraging the development of mathematical thinking. Teachers also used questions as a scaffold for all students so that they could articulate their current understandings, build onto their prior knowledge, and be challenged into their Zone of Proximal Development (Vygotsky, 1978).

Teachers' use of growth points as a conceptual framework

Working with students

All 3 teachers used the conceptual framework of Growth Points to plan units of work, design investigations/tasks and assess students' understandings and growth. They utilised the Growth Points when analysing and assessing students' mathematical thinking through their work samples and during conferences. The framework supported the teachers to know what to listen for, what questions to ask that would support the students to make connections to their prior knowledge, to build on to their prior knowledge, or to challenge their thinking to a higher level.

The framework provided a structure for the teacher to be aware of the different thinking and pathways students could use as they tried to solve problems or to make connections between their prior knowledge and the mathematical concept being investigated. The Growth Points therefore provided the teacher with a pedagogical translation of students' mathematical thinking. They helped teachers identify the conceptual issues confronting students, enabling them to support the students to grasp the big ideas in measurement. In their discussion about Richard Skemp's work, Stacey and MacGregor (2002) concluded that 'teachers who understand about students' thinking teach differently and more effectively than teachers who do not'. Carpenter et al. (1989) assert that an understanding of both the mathematical content and students' thinking is required for effective teaching to happen. The Growth Points provided a structure for this understanding to occur.

Implications for teachers

The commonalities between the mathematics classrooms of Sylvia, Tania and Zoë are both illuminating and instructive. When combined with the weight of complementary research into the complexities of the constructivist classroom, there is much that can be drawn from them that has pragmatic value for all teachers of mathematics. This is the focus of the final chapter.



Implications for Mathematics Teaching and Numeracy Outcomes

This final chapter draws on the combined case analysis to propose a range of mathematics teaching strategies that seem to be integral to optimal levels of numeracy outcomes for all students, including those with special needs and LBOTE students.

Physical and social settings

The research suggests that the physical and social settings of the classroom play a major role in supporting students as they construct understandings in mathematics and numeracy. As illustrated in the combined case analysis, the relationships between the teachers and their students were warm and respectful, and were characterised by a willingness by students and teachers to observe the classroom protocols and to work purposefully together. The organisation of the physical setting of the classrooms was integral to this outcome.

Designing the physical setting

Classroom layout

The physical organisation of the classrooms supported the students' to think and work mathematically and was significant in facilitating collaborative working relationships and a welcoming environment. The tables, for example, were organised in groups, which enabled the students to work together and to support each other's thinking as they built a shared understanding of the mathematical concepts.

Displays

Students and teachers, through brainstorming, actively developed displays of associated language and mathematical terms. These displays were dynamic throughout the unit of work – students and teachers continually added to their lists as new words came up through their investigations. These displays were used by the teachers as a reference source throughout investigations and conferences. The use of displays supported the students to link their informal/prior language to the relevant mathematical terms. Students' work samples were also displayed which provided examples of different ways of thinking and doing the tasks and gave students a sense of being valued.

Manipulative resources

Having ready access to range of manipulatives and the freedom to make their own selections supported students' learning, confidence and their ability to make their own decisions. Students were then able to move more easily from the concrete to the abstract at a pace that suited their thinking.

Creating a supportive social environment

Grouping strategies

Within the constructivist paradigm that operated in each of the three classrooms, mixed ability grouping supported all students, including the LBOTE and special needs students, to work together and share the responsibility of constructing their own understandings in mathematics. Mixed ability grouping and the class protocols (see below) created opportunities for students to learn from each other and to support and respect each other. As the students worked together to construct meaning within this collaborative environment, the teacher was free to support individual students through conferencing.

As respect of each other's ideas was encouraged, students who were usually quiet and considered to be of low ability gradually started to contribute to group discussions.



The change in these students' attitude and confidence seems to have resulted from the combination of classroom norms and the immediate feedback they received, initially from the teacher, and later from other group members.

Protocols – encouraged behaviours in the classroom

The protocols that existed in all 3 classrooms valued the students' thinking and encouraged students and teacher to interact and work together to build understandings in mathematics. Each teacher operated on the belief that all students could learn and work mathematically. Students were expected to investigate and to record their own thinking. One of the protocols that contributed to the success of the interactions was the practice of expecting students who were experiencing difficulties to consult with other group members before indicating that they required teacher assistance. This classroom protocol effectively freed the teacher to focus on individual students or small groups of students, involving them in interactions that intended to build understandings in mathematics/numeracy.

Students' working styles

The co-independent style of working supported students to work on their own task using their own ideas and strategies within a group setting. Within this group they discussed and shared their ideas and strategies or sought support from other group members when required. However, they always recorded their thinking in their own way. Through this, students grew more confident about the construction of their own knowledge. This method of working seemed to have assisted most students as they constructed their own meaning within a learning enriched environment.

The mathematical experience

Supportive conferencing

Conferences provided students with quality time with the teacher either individually or in a small group of students. During this time each of the three teachers focused on individual students' needs and understandings, responding to different pathways students took when investigating, even when the students were constructing knowledge around the same mathematical ideas. The conferences provided the teachers with opportunities to address difficulties the students might be facing, as well as enabling them to assess, support, challenge and affirm individual students' thinking. In order to ensure equitable access to the teacher's time, the teachers monitored who they conferenced in a lesson and, over a few lessons, ensured that all students were engaged in at least one conference.

When teachers focused on individual students' thinking and understandings, all students, regardless of whether they were LBOTE, special needs or high achieving students, responded well, and as documented in the qualitative data their academic performance improved. In a one-to-one interaction, these teachers were able to access the form of the students' mathematical thinking, which assisted them in providing the necessary scaffolding to support individual students towards desired results. Indirectly, this legitimised the position that there may be more than one way of solving the same problem, which could influence the development of flexible pathways in problem solving.

Conferencing supported the development of 'common knowledge' between the teacher and the student. After a conference, students were often seen to share their thinking with others, which in turn served as a means for the students to consolidate their thinking, understandings and the

sharing of common knowledge. Rittenhouse (1998) reports that students' performances improve when they spend time interacting with the teacher.

In this research the conferencing that was supportive of individual student learning had two interactive phases. Figure 48 illustrates *Supportive conferencing,* which consists of 2 interactive phases: reconnaissance and scaffolding.

Phase 1: Reconnaissance

In the reconnaissance phase, the teacher established each student's current thinking and understanding, through a process of observation, analysis of work and asking reconnaissance questions. During this phase the teacher reviewed the student's thinking within a conceptual framework called *Possible Learning Connections* to establish possible links the student may need to build further understanding.

Phase 2: Scaffolding

Once a teacher had established a student's prior or informal knowledge, the student was supported in linking this knowledge to the mathematical concept under investigation. During this phase many different pathways could be taken. The teacher facilitated the pathway in response to the connections individual students made as they built mathematical understanding.

The teacher either challenged the student to build onto their existing knowledge and or to refine the strategies being used in the investigation by asking linking or challenging questions. The teacher also used the information gained during reconnaissance to decide whether to redirect the student's thinking either by asking questions or introducing a new task. The teacher provided direct information when she thought it was needed for the student to be able to continue investigations.

The phases of a conference were not exclusive but rather mutually inter-related and the teacher strategies often moved from one phase to the other in response to the directions the student was taking. Supportive conferences gave teachers insight into their students' thinking and understandings. As a result, the teachers were better able to respond to individual students' needs.

In both phase 1 and 2 the teachers employed a range of different types of questions for different purposes that were supportive of student learning. These questions have been categorised as *reconnaissance questions, linking questions and challenging questions.*

Reconnaissance questions

The reconnaissance questions required the students to verbalise their thinking and understandings. These questions had several purposes. First, they enabled the teacher to ascertain what the student knew and understood. Second, when verbalising their thinking the students were required to organise their thinking into a logical form, hence forcing them to clarify their thoughts. These questions scaffolded the students' communication of ideas and encouraged the development of metacognition (Schoenfeld, 1992). Third, because the students were invited to communicate their thinking, they were more conscious of their knowledge and as a result they were willing to share it with other group members, which may have contributed to increasing their confidence (see, for example, Skemp, 1986).

Linking questions

The linking questions assisted students to connect their existing knowledge (prior mathematical knowledge, informal knowledge, language, numeracy and their experiences) to the mathematical concept being investigated or to the problem they were attempting to solve.

Challenging questions

Challenging questions forced students to delve deeper into their understandings, thereby forcing them to reflect on what they already knew and to move onto a higher level of thinking. A challenging question could also place a student's current thinking into conflict. This gave the student opportunities to self-correct and re-address previous understandings as the student assimilated and adapted to the current conceptual understandings.

Encouraging students to verbalise and record their thinking

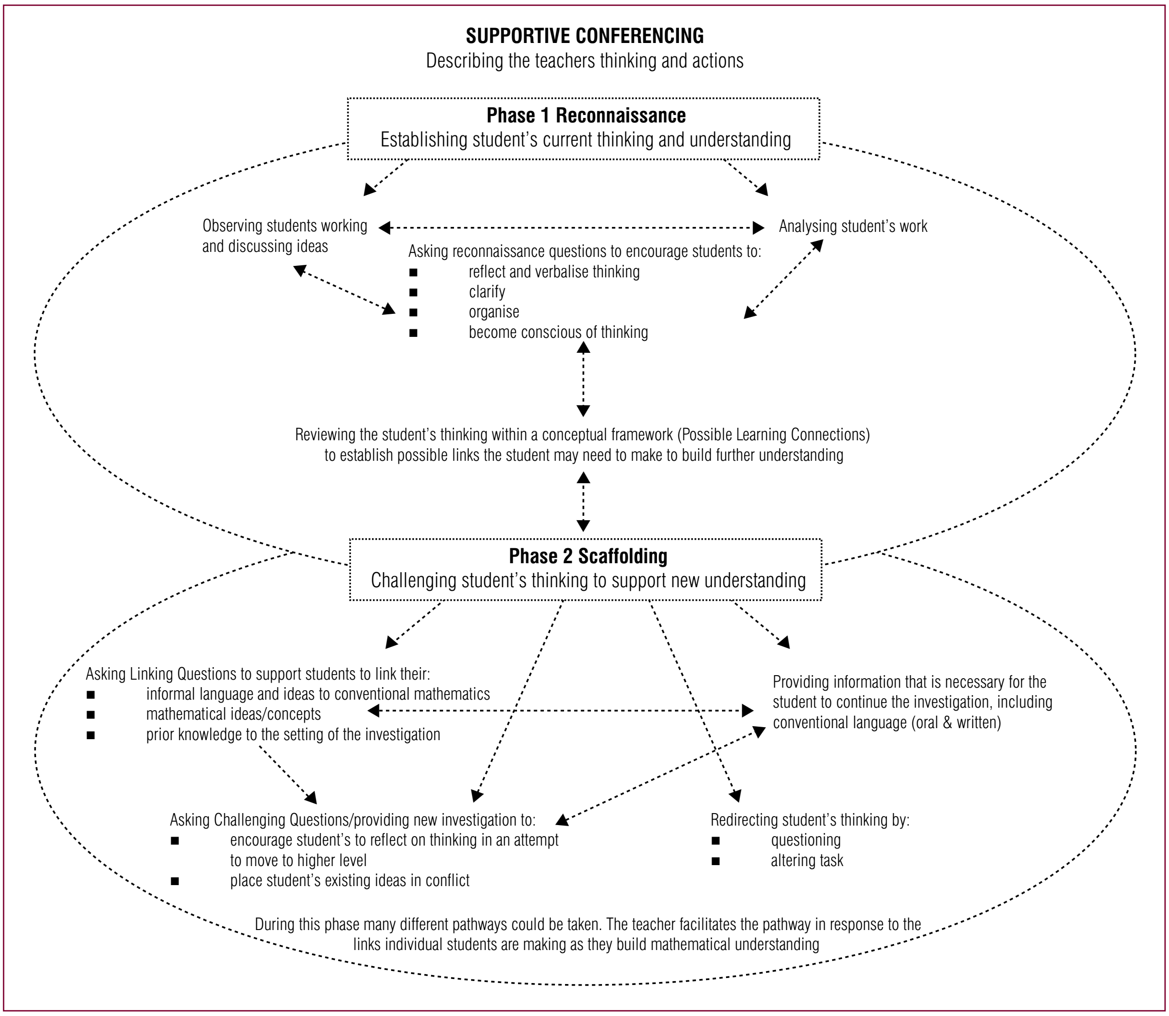
It was evident in the classrooms studied that the students seemed comfortable in talking about their thinking. They did not seem inhibited by what others might say or do as they engaged in the construction of their own understandings. Other research suggests that verbalising their thinking assists students to create a network of ideas (Hiebert & Carpenter, 1992) but the current research goes further; it suggests that such collaborations contributed to students' confidence, not only in mathematics, but also in other subject areas. As well as verbalising their thinking the students were asked to record their thinking in their own way. This not only supported the consolidation of their learning but it also provided the teacher with access to the students' thinking/learning during or after the lesson.

However, although the teachers supported the students in verbalising their understanding and linking their present mathematical thinking to their prior and informal knowledge, sometimes teachers did not succeed in challenging students to a higher level of thinking. This happened when teachers asked closed questions where there was only one response pathway and hence one outcome. On the other hand, when they recognised the connections students could make, they were in a better position to build onto these connections and thus enhance students' mathematical understandings.

Encouraging students to work mathematically

During a conference the teacher encouraged students to work mathematically, collecting and searching for patterns in their data, making conjectures and drawing conclusions and generalisations. At the end of a conference the teacher tended to summarise the crucial parts of the discussion using mathematical terms, indicating those aspects that were critical in the construction of understanding (shared knowledge). Such emphasis on a mathematical way of thinking engaged students in searching for and conjecturing about their own solutions. This kept them interested and motivated as well as arousing in them a sense of curiosity so that they were meaningfully engaged. The students appeared confident and proud to talk to about what they were doing. This could be an indicator of growing ownership and confidence in thinking and working mathematically.

Figure 48. Supportive conferencing



Open investigative tasks

The mathematical tasks/investigations were another factor that seemed to have been effective in supporting the students to work mathematically and to develop mathematical understanding. The teachers set investigative and open-ended tasks, allowing the students to use their prior/informal knowledge and experiences. As a result, all students were able to begin at their own level. The students' thinking and strategies were later challenged as the teacher supported them to build on their prior knowledge. The investigative nature of the tasks enabled the students to collect data and searching for patterns in their data as they made conjectures and generalisations.

Teachers designed and adapted investigative and open-ended tasks based on their understanding of each student's thinking and interests gained through the conference process. This ensured that all students were engaged in the tasks. The students were also confident, knowing that they had each other's support, as well as that of the teacher, to affirm and challenge their thinking. As assessments were ongoing, students received continuous feedback from the teacher.



Accessing resources

Providing students with a range of resources appears to have supported them during the investigations. The students had to make decisions about the resources they wanted to use and how they used them. Students tended to choose resources they were familiar with, suggesting a link with their prior knowledge. Being able to choose from a large selection of resources assisted the teacher in accessing the students' thinking. For example, students were given a range of units when measuring, both non-standard and standard. This not only allowed the students to start from their own knowledge base, but also enabled the teacher to gain insights into the students' understandings of measurement units, for example, did they use circular counters to measure area, thereby ignoring the gaps?

By being able to enter the task at their own level and having the resources to model their thinking, students were able to investigate and build on their current understanding of the concepts. Students were given many opportunities to make decisions about their learning. It was apparent towards the end of the year that students often discussed and justified the choice of resources with their peers, and occasionally they chose to use resources that the teacher had not provided.

Moving from growth points to possible learning connections

During the reconnaissance phase of this research, the teachers were introduced to the pedagogical concept of Growth Points. Evidence from this research project has illustrated that the Growth Points concept assisted the teachers in understanding the array of connections students might take as they make mathematical sense of linear measurement and area. The concept of Growth Points also provided a framework on which the teachers could base their on-going assessment of students' thinking and understanding during conferences, and their support of students' construction of understanding in measurement. Growth Points also provided a supportive framework for the teacher when observing and assessing students' understandings, identifying connections they are making, and subsequently to formulate questions to support individual needs and growth.

However, during the project analysis, we realised that the Growth Points were not necessarily being seen as describing the different pathways students went along as they constructed their mathematical understandings, but rather as a set of outcomes for all students to pass through. The name Growth Points can be misleading, and in uncovering the mixed messages the name has incited, a re-naming is proposed: A web of *Possible Learning Connections* more accurately describes the conceptual framework for assessing and conferencing students' learning.

Possible learning connections

A web of *possible learning connections* encapsulates the network of some possible connections that students may make as they construct and make meaning of the unifying ideas of measurement. These connections form the conceptual understanding behind the unifying ideas. This concept builds on the Hiebert and Carpenter's definition of understanding in terms of a 'web'-like network of interconnected ideas. 'All nodes in the web are ultimately connected, making it possible to travel between them by following established connections' (Hiebert & Carpenter, 1992, p. 67).

The web of *possible learning connections* assists the teacher to analyse what connections the student has made and how strong those connections are, enabling the teacher to design tasks and pose possible questions to further challenge the students' thinking. It also assists the teacher to identify,

acknowledge and value students' existing thinking and knowledge, prior to challenging the students to their zones of proximal development (Vygotsky, 1978).

With the central focus on the unifying ideas of measurement rather than on individual attributes, students would be expected to build knowledge that could support them in their future learning. For example, a new measuring system could be investigated using their understanding of the unifying ideas of measurement units. While we acknowledge that the concept of *possible learning connections* is far from complete, it is the beginning for future studies.

Assessment

These teachers employed a range of ongoing assessment strategies that provided the students with immediate feedback, as well as informing the ongoing facilitation and learning program. These strategies included oral assessment during conferences, work sample analysis, and observations. Assessment in the form of work sample analysis was carried out both within and after each session to inform the next session. The practical observation test and the pencil-and-paper test administered at the beginning and the end of the research were instrumental in informing the teacher of the students' current knowledge.

Implications for schools

This research illustrates that professional learning through action research supported the teachers to further enhance their knowledge of mathematics, their understanding of students' thinking and the ways students can learn using constructivist pedagogy. It also showed that when teachers worked with supportive colleagues – researchers, consultants, or other teachers with similar interests and goals – they were better placed to reflect on their students' thinking. All the teachers in this research felt that the reflective moments following their classroom observations, as well as the collaborative sessions they engaged in with other teachers and project researchers, contributed to their outlook on teaching and learning mathematics**.**

Following this research schools may wish to encourage small groups of teachers within a band (SACSA) or year level to use an action research methodology to reflect on their teaching and on the students' learning. Outside input during theoretical discussions and the reconnaissance stage could further assist deep reflections. Schools and teachers may wish to form hub groups where together, through action research, they could have opportunities to further reflect on their pedagogy in relation to students' thinking and learning.

The group of teachers in this research met on a regular basis to collaborate and share findings, reflect on observations and students' work samples and discuss the next plans of action. Collaboration is a key component of the action research methodology; teachers not only need to share the same interest and goals but also willingness to collaborate and reflect. Action researching their own pedagogy within their own school settings could assist teachers to be more reflective and would further empower them. As with the research group most of the participating teachers were eager to investigate their pedagogy and to read about other research and theories within their area of interest.

Teachers also need to have on-going input from consultants and on-going support, with regular workshops and classroom support over a period of time. Using the same principle as with students' learning, consultants need to work with the teachers, addressing their individual needs. During the workshops teachers might benefit from working together in small groups. This approach seems to be preferred by the teachers involved in this project as well as those from other projects, rather than one-off intensive professional development.

Possible further research

The new understandings and implications mentioned that have arisen from this research have provided a range of constructive insights into how teaching and working mathematically can be enhanced. However, like any piece of research, we have identified a number of limitations. There is significant scope for further research. Following is a brief proposal for where this research could be extended.

We observed that students who were quiet and/or low achievers increased their level of performance. Their teachers and their parents all reported that they had seen marked changes in the students' performances and confidence in mathematics, as well as in other areas. However, this research did not allow us to investigate whether the increased confidence of the students was due to the strategies that the teachers put in place, or due to some other factor(s). Further research is therefore needed in this area.

Another interesting aspect that we observed was the relationship between the teacher and the students. This relationship involved the teachers' valuing of the students' prior and informal knowledge and the friendliness of the teacher when working with the students. Observing this relationship, we wondered in what forms student–student and student–teacher relationships (for example, emotional, gender, ability) contributed positively to a student's performance.

Due to the time constraints on this study, we were unable to address a number of factors that we felt could have contributed to the significant growth of each student's understanding and performance in mathematics, for example, students' use of mathematics outside the mathematics classroom. When students were asked to apply their mathematical knowledge to solve contextualised problems, some students experienced difficulty when dealing with numbers and some found it hard to interpret their findings but their ability improved with practice. Anecdotal accounts indicate that some students were using the mathematics outside the mathematics classrooms and that their confidence had grown.

The teachers benefited from using the web of *possible learning connections* as a conceptual framework when planning units of work, assessing students' thinking and recognising the different connections students may take. By recognising and understanding the diversity in their students' thinking, teachers are more able to respond appropriately. While it is acknowledged that the web of possible connections was found to be integral in supporting teachers to scaffold student learning, there needs to be further research involving teachers in classrooms developing such conceptual frameworks across other areas of mathematics and across curriculum.

Summary

In an attempt to make sense of the complexity of the constructivist mathematics classroom this research has found that there is interplay between the physical setting of the classroom and a social setting that nurtures productive relationships between students and their teacher. Added to this is the form of the mathematical experience and the integral role that small group investigations, conferencing, and the framework of possible learning connections play, in supporting students to construct meaningful mathematical understandings.

It is plain that in this dynamic classroom complexity, it is the teacher's professional judgment that will always be central to engaging and extending students' mathematical education along the pathways to powerful numeracy outcomes.

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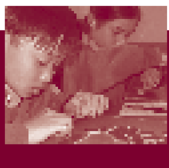


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